

Probabilistic techniques - tutorials

Classwork 2 – Expectations and method of alterations

1. Show that there is a two coloring of the edges of K_n with at most $\binom{n}{a} 2^{1-\binom{a}{2}}$ monochromatic K_a .
2. Let M be an $n \times n$ matrix with entries uniformly independent chosen from $\{-1, 1\}$. Determine $\mathbb{E}[\det(M)]$.
3. Let $n \geq 2$, $H = (V, E)$ an n -uniform hypergraph with $|E| = 4^{n-1}$ edges. Show that there is a coloring of V by four colors such that no edge is monochromatic.
4. Denote by $R(\cdot, \cdot)$ the Ramsey numbers. Fix $k \in \mathbb{N}$. Prove that for any integer n , it holds that $R(k, k) > n - \binom{n}{k} 2^{1-\binom{k}{2}}$. Can you write the expression on the right hand side in terms of k ?
5. Fix a graph F and define $\text{ex}(n, F)$ to be the maximal number of edges of an n -vertex graph that does not contain F as a subgraph. Show that for any F on $k > 2$ vertices with at least $2k - 3$ edges it holds $\text{ex}(n, F) = \Omega(n^{3/2})$.
6. A dominating set of an undirected graph $G = (V, E)$ is a set $U \subseteq V$ such that every vertex $v \in V \setminus U$ has at least one neighbor in U . Let $G = (V, E)$ be a graph on n vertices, with minimum degree $\delta > 1$. Then G has a dominating set of at most $n \frac{1+\ln(\delta+1)}{\delta+1}$ vertices.
7. Let $G = (V, E)$ be a graph on n vertices with minimum degree $\delta > 1$. Prove that there is a partition of V into two disjoint sets A and B such that $|A| \leq \mathcal{O}(n \frac{\ln(\delta)}{\delta})$ and each vertex in B has at least one neighbor in A and at least one neighbor in B .