9. cvičení z PSt

1

Máme tvar

$$P(|X - \mathbb{E}X| \geq t\sigma_X) \leq \frac{1}{t^2}.$$

Položme $t = a/\sigma_X$. Pak

$$P(|X - \mathbb{E}X| \geq a) \leq \frac{\mathrm{var}(X)}{a^2}.$$

2

Nechť $X \sim \mathrm{Bin}(n, 1/3)$, tedy $\mathbb{E}X = n/3$, $\mathrm{var}(X) = \frac{2n}{9}$.

(a) Markov:

$$P(X \geq n/2) \leq \frac{\mathbb{E}X}{n/2} = \frac{n/3}{n/2} = \frac{2}{3}.$$

(b) Čebyšev:

$$P(X \geq n/2) \leq P(|X - n/3| \geq n/6) \leq \frac{2n/9}{(n/6)^2} = \frac{8}{n}.$$

(c) Z binomického rozdělení:

$$P(X \geq n/2) = \sum_{k=\lceil n/2 \rceil}^{n} \binom{n}{k} \left(\frac{1}{3}\right)^k \left(\frac{2}{3}\right)^{n-k}.$$

3

Máme $\mathrm{var}(X_i) \leq 1$, tedy $\mathrm{var}(\bar{X}_n) = \frac{1}{n}$ (díky nezávislosti).

(a)

$$\sqrt{\mathrm{var}(\bar{X}_n)} = \frac{1}{\sqrt{n}} \leq 0.01 \;\Rightarrow\; n \geq 10^4.$$

(b)

$$P(|\bar{X}_n - h| \geq 0.05) \leq \frac{1}{n(0.05)^2} = \frac{400}{n} \leq 0.01 \;\Rightarrow\; n \geq 40000.$$

4

(a)

$$\mathbb{E}X_i = \frac{\pi}{4}, \quad \mathrm{var}(X_i) = \frac{\pi}{4}\left(1 - \frac{\pi}{4}\right).$$

(b)

$$\mathbb{E}\bar{X}_n = \frac{\pi}{4}, \quad \mathrm{var}(\bar{X}_n) = \frac{1}{n}\frac{\pi}{4}\left(1 - \frac{\pi}{4}\right).$$

(c)

$$\bar{X}_n \xrightarrow{P} \frac{\pi}{4}, \quad \bar{X}_n \xrightarrow{s.j.} \frac{\pi}{4}.$$

(d)

$$\mathrm{sd}(\bar{X}_n) \sim \frac{c}{\sqrt{n}} \Rightarrow n \sim 10^{2k} \text{ pro } k \text{ desetinných míst.}$$

(e)

$$\mathbb{E}Y_i = \frac{\pi}{4}, \quad \operatorname{var}(Y_i) = \mathbb{E}(1 - U_i^2) - \left(\frac{\pi}{4}\right)^2 = \frac{2}{3} - \frac{\pi^2}{16}.$$

$$\operatorname{var}(\bar{Y}_n) = \frac{1}{n}\left(\frac{2}{3} - \frac{\pi^2}{16}\right).$$

(f) Menší rozptyl $\Rightarrow$ metoda s Y_i je přesnější.