HW7

November 27, 2024

Name: _____

- Let G with minimum degree $n/2$ be a bipartite graph with $2n$ vertices such that each of its classes of bipartition has n vertices. Prove that G must contain a perfect matching.

- Consider a logical formula of the form $(x_1 \vee \neg x_2 \vee \cdots) \wedge (x_3 \vee \cdots) \wedge \cdots$, i.e. one that is a conjunction of clauses, which are disjunctions of literals, and each literal is either a variable or its negation. A formula is satisfiable if it is possible to substitute true/false for the variables so that the whole formula is true.

  Prove that any formula whose each clause contains exactly 3 literals and each variable occurs in just 3 different clauses is satisfiable.

  Example:

$$(x_1 \vee \neg x_2 \vee x_3) \wedge (\neg x_1 \vee x_2 \vee x_3) \wedge (\neg x_1 \vee \neg x_2 \vee \neg x_3)$$

$$x_1 = True,\ x_2 = False,\ x_3 = True$$