HW4

November 11, 2024

Name: ____

Finite projective plane:
The pair $(X, \mathcal{P})$ where X is a finite set ("points") and $\mathcal{P} \subseteq 2^X$ is a collection of its subsets ("lines"), such that:

- **(A0)** $(\exists C \subseteq X, |C| = 4)$: $\forall P \in \mathcal{P}$: $|C \cap P| \leq 2$ (There exist 4 points in general position).
- **(A1)** $\forall x \neq y \in X$ $(\exists! P \in \mathcal{P})$: $x, y \in P$ (Every pair of points is contained in exactly one line).
- **(A2)** $\forall P \neq Q \in \mathcal{P}$: $|P \cap Q| = 1$ (Every pair of lines intersects in exactly one point).

1. Give two examples of a pair $(X, \mathcal{P})$ that satisfies axioms A1 and A2, but not axiom A0.
2. For an infinite number of values of n, construct a graph on n vertices with $\Omega(n^{3/2})$ edges, which does not contain a 4-cycle($K_{2,2}$) as a subgraph.
3. **(5 additional points)** Determine all the pairs $(X, \mathcal{P})$ that satisfies axioms A1 and A2, but not axiom A0.