## Topological methods in combinatorics

Class work – Homology

09.05.2024

- 1. Consider a graph G as a simplicial complex. Interpret the meaning of 0,1-cycles and boundaries.
- 2. Compute the homology groups of  $\Delta_d$  from definition.
- 3. Compute the homology groups of  $\Delta_d$  using homotopy invariance.
- 4. We have for  $d \ge 1$  that

$$H_n(S^d) \cong \begin{cases} \mathbb{Z} & \text{if } n = 0, d, \\ 0 & \text{otherwise.} \end{cases}$$

Prove that there is no retraction from  $B^d$  to  $S^{d-1}$ . (Do you see that this implies Brouwer's theorem?)