Úlohy ke cvičení

Úloha 1: Describe all solutions of the following system and verify them.

a)
$$
\begin{aligned}
-x_1 &+& x_2 &-& 3x_3 &+& 4x_4 &=& 1 \\
2x_1 &-& x_2 &+& 4x_3 &-& 7x_4 &=& 0 \\
-x_1 &-& x_2 &+& x_3 &+& 2x_4 &=& -3 \\
-x_1 &+& 2x_2 &-& 5x_3 &+& 5x_4 &=& 3
\end{aligned}
$$

Úloha 2: Solve the system with augmented matrix

$$
\left(\begin{array}{cccc|c}
a & 1 & 1 & 1 & 1 \\
1 & a & 1 & 1 & 1 \\
1 & 1 & a & 1 & 1 \\
1 & 1 & 1 & a & 1
\end{array}\right)
$$

depending on the parameter a.

Úloha 3: Prove or disprove, whether for matrices $\mathbf{A}, \mathbf{B}, \mathbf{C}$ and $\mathbf{0}$ of the same type and real numbers α, β holds:

a) $\mathbf{A} + (\mathbf{B} + \mathbf{C}) = (\mathbf{A} + \mathbf{B}) + \mathbf{C}$

b) $\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$

c) $\mathbf{A} + \mathbf{0} = \mathbf{A}$

d) $\alpha(\beta\mathbf{A}) = (\alpha\beta)\mathbf{A}$

e) $\alpha(\beta\mathbf{A}) = \beta(\alpha\mathbf{A})$

f) $\mathbf{A} + (-1)\mathbf{A} = \mathbf{0}$

g) $1\mathbf{A} = \mathbf{A}$

h) $\alpha(\mathbf{A} + \mathbf{B}) = \alpha\mathbf{A} + \alpha\mathbf{B}$

i) $(\alpha + \beta)\mathbf{A} = \alpha\mathbf{A} + \beta\mathbf{A}$

j) $\alpha\mathbf{A} + \beta\mathbf{B} = (\alpha + \beta)(\mathbf{A} + \mathbf{B})$

k) $(\mathbf{A}^T)^T = \mathbf{A}$

l) $(\mathbf{A} + \mathbf{B})^T = \mathbf{A}^T + \mathbf{B}^T$

m) $(\alpha\mathbf{A})^T = \alpha(\mathbf{A}^T)$

Úloha 4: Split the matrix

$$
\begin{pmatrix}
0 & 1 & 2 & 3 \\
3 & 2 & 1 & 4 \\
4 & 3 & 0 & 1 \\
3 & 0 & -1 & 2
\end{pmatrix}
$$

into a sum of two matrices, where one is symmetric (i.e. $\mathbf{A} = \mathbf{A}^T$) and the other antisymmetric (i.e. $\mathbf{A} = -\mathbf{A}^T$).