

Discrete Mathematics

Exercise sheet 1

3 /6 October 2016

1. Prove the following statements by mathematical induction:

(a) $\sum_{i=1}^n (2i-1) = n^2$.

(b) $6n^2 + 2n$ is a multiple of 4.

(c) $\prod_{i=2}^n \frac{i-1}{i} = \frac{1}{n}$.

(d) $\sum_{i=1}^n i^2 = \frac{1}{3}n^3 + \frac{1}{2}n^2 + \frac{1}{6}n$.

Try to give an alternative proof for each of (a), (b) and (c) that does not use induction.

2. Prove the following statements using the method of proof by contradiction:

(a) There is no largest natural number.

(b) If n^2 is an odd number then n is odd.

(c) If a, b and c are natural numbers such that $a^2 + b^2 = c^2$ then either a is even or b is even.

3. Let F_n be the n th Fibonacci number, defined by $F_0 = 0$, $F_1 = 1$ and for $n \geq 1$ by the recurrence $F_{n+1} = F_n + F_{n-1}$. Prove the following statements by mathematical induction:

(a) $\sum_{i=0}^n F_i = F_{n+2} - 1$.

(b) $\sum_{i=0}^n F_i^2 = F_n F_{n+1}$.

(c) F_{3n} is even

(d) $F_{n-1}F_{n+1} - F_n^2 = (-1)^n$ (for $n \geq 1$).

4. You are at the bottom of a flight of stairs with n steps that you can ascend by taking one or two steps at a time. How many ways can you go up? (*If $n = 2$, you have 2 choices: take one step twice in a row, or two steps in one go. If $n = 3$, you have 3 choices: three single steps, or one single followed by one double, or one double followed by one single.*)

In order to get to S11 for class you ascend two flights of stairs from the ground floor to the first floor, each flight consisting of 18 steps. How many times would you need to go upstairs from ground floor to first floor in order to exhaust all possible ways of going up 18 stairs?