Interpretations – how to speak about sparsity in dense graphs

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Prague

The plan

Today:

- Part I: Introduction and motivation
- Part II: More about interpretations
- Part III: Interpretations and sparsity – overview

Main goal of today's lecture: to provide you with a working knowledge of interpretations

Next lecture (Friday 30.11.): applications and recent results

Part I: Introduction and motivation

What do we really study and why?

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Different perspective/motivation: We understand sparse graphs quite well now and want to extend these results to dense graphs.

Interpretations

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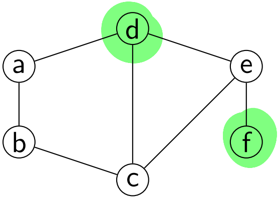
(Much) more general construction – interpretations.

Interpretations

$$\psi(x, y) \equiv x \neq y \wedge \exists z : (z \neq x) \wedge (z \neq y) \wedge \mathrm{edge}(x, z) \wedge \mathrm{edge}(z, y)$$



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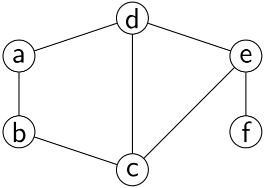

Pairs of vertices satisfying $\psi(x, y)$:
$\{a, c\}, \{b, d\}, \{c, d\}, \{c, e\}, \{d, e\}, \{d, f\}, \{a, e\}, \{b, e\}, \{c, f\}$

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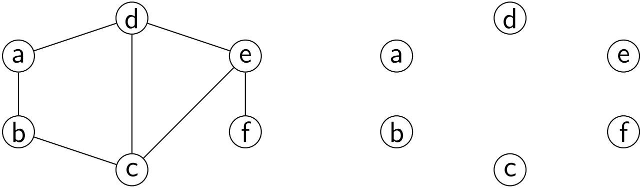

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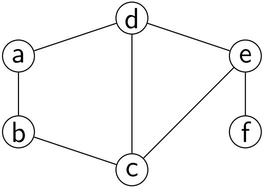

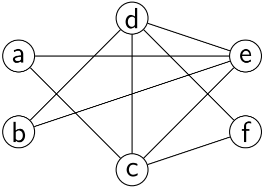

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Interpretations

Consider a formula $\psi(x, y)$ which is irreflexive and symmetric.

From a graph G, this formula "creates" graph $H = I_\psi(G)$ as follows:

$$V(H) = V(G)$$

$$E(H) = \{\{u, v\} \mid G \models \psi(u, v)\}$$

We then say that H is interpreted in G using ψ.

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This is easily extended to graph classes:

$$I_\psi(\mathcal{C}) = \{H \mid H = I_\psi(G) \text{ for some } G \in \mathcal{C}\}$$

We say that class $\mathcal{D}$ of graphs is interpretable in $\mathcal{C}$ if there exists $\psi(x, y)$ such that $\mathcal{D} \subseteq I_\psi(\mathcal{C})$

Notes

- This definition of interpretation is:
  - tailored to graphs
  - simplified (normally $V(H) \subseteq V(G)$)
- Whenever we speak about graphs G and H, graph H is the result of an interpretation.
- All graphs/structures in this talk are finite

Example

The class of all complete bipartite graphs is interpretable in the class of all stars forests

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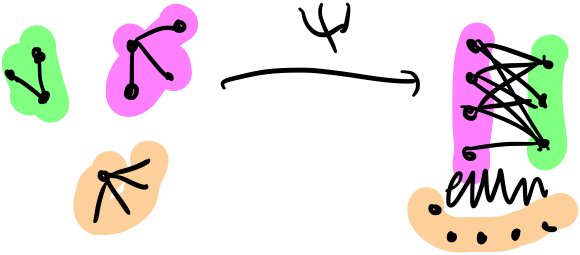
The class of all complete bipartite graphs is interpretable in the class of all stars forests

What we need to do: Write down a formula $\psi(x,y)$ such that for every complete bipartite graph H there exists a star forest G such that $H = I_\psi(G)$

$$G \xrightarrow{\psi} H$$

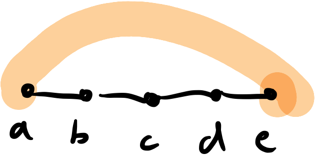
$$\psi(x,y) := x \neq y \wedge \neg\left(E(x,y) \vee \exists z\,(E(x,z) \wedge E(z,y))\right)$$

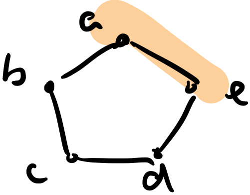


Example

The class of all cycles is interpretable in the class of all paths.

$$\psi(x,y) := (x \neq y) \wedge \Big(E(x,y) \vee \big(\deg(x)=1 \wedge \deg(y)=1\big)\Big)$$





Interpretations – examples

$\psi(x, y)$ can be:

- $x \neq y$ – creates a clique out of any graph
- $x = y \wedge x \neq y$ – creates an empty graph out of any graph
- $x \neq y \wedge \neg E(x, y)$ – creates the edge-complement of any graph

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- $x \neq y \wedge \neg E(x, y)$ – creates the edge-complement of any graph
- There is a path of length at most 3 between x and y.
- There is a path of length at most 5 between x and y such that there are two vertices of degree 8 on it.
- Vertex x has degree 3 and vertex y has a neighbor with degree at least 5.
- both x and y have degree at least 7
- ...

Why are interpertations good?

Let $\varphi :=$ "there exists a dominating set of size 3" and let H be a (dense) graph.

We want to know whether

$$H \models \varphi$$

Let $\psi(x, y)$ be an interpretation formula saying "the distance between x and y is at most 2" and let G be a (sparse) graph such that

$$H = I_\psi(G)$$

The key observation:

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The key observation:Instead of asking

"Does H have a dominating set of size 3?"

we may as well ask

"Does G have 3 vertices such that all vertices are at at distance at most two from them?"

Why are interpertations good?

An interpretation...

- ...assigns to every graph G a new graph H...
- ... but also allows us to translate (reduce) every formula φ about H to a formula φ' about G such that

$$H \models \varphi \iff G \models \varphi'$$

$$\begin{tikzcd} G \arrow[r, "\psi"] & H \\ \varphi' & \varphi \arrow[l, "\psi"'] \end{tikzcd}$$

$$\varphi = \exists x_1 \forall x_2 \dots E(x_1, x_2) \dots E(x_3, x_2) \dots$$

$$\varphi' = \exists x_1 \forall x_2 \dots \psi(x_1, x_2) \dots \psi(x_3, x_2) \dots$$

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Consequence:
Let $\mathcal{D}$ be a class of (dense) graphs and assume that $\mathcal{D} = I_\psi(\mathcal{C})$ for some (sparse) graph class $\mathcal{C}$.
The study of FO logic on $\mathcal{D}$ can be reduced to the study of FO logic on $\mathcal{C}$.

Interpretations – algorithmic perspective

Let $\mathcal{D}$ be a class of (dense) graphs and assume that $\mathcal{D} = I_\psi(\mathcal{C})$ for some (sparse) graph class $\mathcal{C}$.

We know that to every $H \in \mathcal{D}$ there exists $G \in \mathcal{C}$ such that

$$H = I(G)$$

Algorithmic problem: Given H, find G.

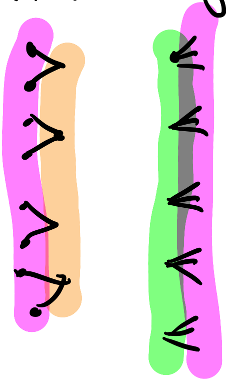
Part II: More about interpretations

Interpretations – examples

Can we interpret the class of all complete bipartite graphs in the class of graphs of degree at most d? $d = 5$

$$\varphi(x,y) := \deg(x)=2 \wedge \deg(y)=3$$

$$\gamma(x) := \deg(x) > 1$$

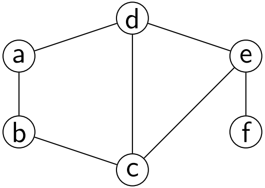


$K_{4,5}$

Interpretations – removing vertices

$\psi(x, y) \equiv x \neq y \wedge \exists z : (z \neq x) \wedge (z \neq y) \wedge \text{edge}(x, z) \wedge \text{edge}(z, y)$

$\nu(x) =$ x has degree at most 2



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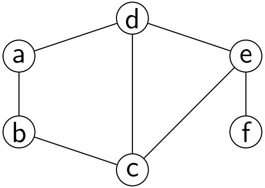

Vertices satisfying $\nu(x)$: a, b, e, f

Pairs of vertices satisfying $\psi(x, y)$: $\{a, c\}$, $\{b, d\}$, $\{c, d\}$, $\{c, e\}$, $\{d, e\}, \{d, f\}, \{a, e\}, \{b, e\}, \{c, f\}$

Interpretations – extension

$$\psi(x, y) \equiv x \neq y \wedge \exists z : (z \neq x) \wedge (z \neq y) \wedge \mathrm{edge}(x, z) \wedge \mathrm{edge}(z, y)$$

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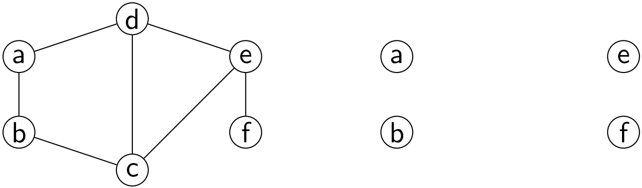

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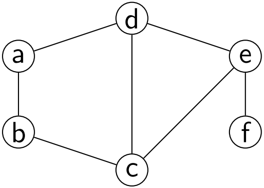

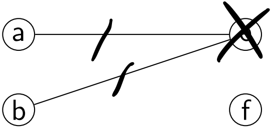

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Interpretations – examples

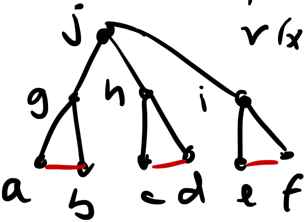
The class of all complete bipartite graphs is interpretable in the class of graphs of degree at most d?

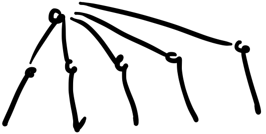
Interpretations – examples

The class of all matchings is interpretable in trees of height 2.

$E(x,y) := x \neq y \wedge dist(x,y) = 2$

$\nu(x) := dg(x) = 1$





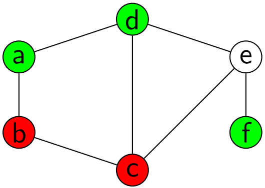
$\psi(x,y) = E(x,y)$

$\nu(x) = dg(x) \leq 2$

Interpretations with colors

If we start with a class of clored/labled graphs, we can use colors/lables in interpretations:

$$\psi(x,y) \equiv x \neq y \wedge \exists z : (z \neq x) \wedge (z \neq y) \wedge \mathsf{edge}(x,z) \wedge \mathsf{edge}(z,y)$$
$$\wedge \left[(\mathsf{green}(x) \wedge \mathsf{red}(y)) \vee (\mathsf{red}(x) \wedge \mathsf{green}(y))\right]$$



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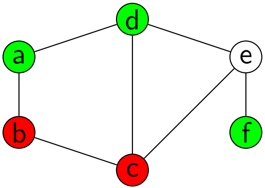

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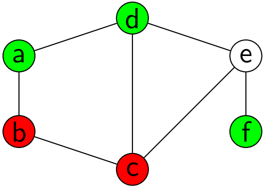

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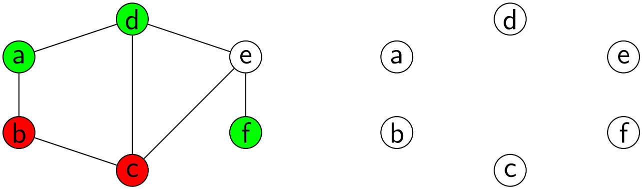

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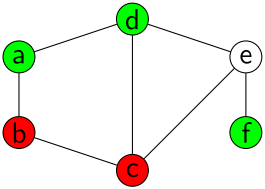

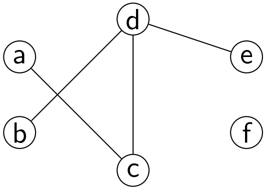

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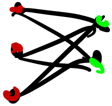
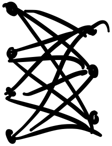
Example

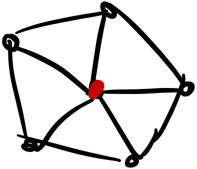
The class of all complete bipartite graphs is interpretable in the class of all colored paths.

$$\psi(x,y) = \mathrm{Red}(x) \wedge \mathrm{Gree}(y)$$

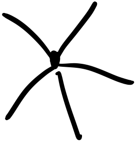




The class of all wheels is interpretable in the class of all colored cycles.

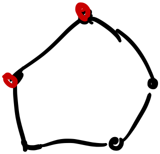


$$\psi(x,y) := (x - y) \wedge \big((\mathrm{Red}(x) \wedge \neg \mathrm{Red}(y)) \vee (\mathrm{Blue}(x) \wedge \mathrm{Blue}(y)) \big)$$



Example

The class of all paths is interpretable in the class of all colored cycles.

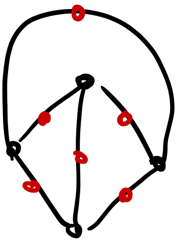


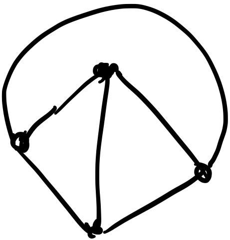
$$\psi(x,y) = E(x,y) \wedge \neg(\mathrm{Red}(x) \wedge \mathrm{Red}(y))$$

Example

The class of all graphs is interpretable in the class of all 2-degenerate graphs.

2-degenerate graph : every subgraph has a vertex of degree ≤ 2



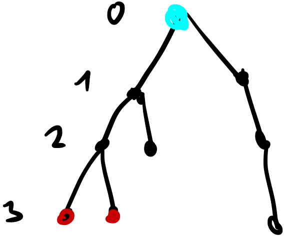


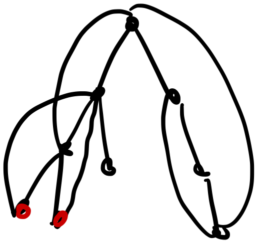
$V(x) = \neg \mathrm{Red}(x)$

$\varphi(x,y) :=$ x and y have a common red neighbor

An important example

Show that for every d there the class of graphs of treedepth at most d is interpretable in colored forests of height d.





$$\psi(x,y) = (x \text{ below } y) \wedge \mathrm{Red}(x) \wedge (\mathrm{Level}(y)=2 \vee \mathrm{Level}(y)=0)$$

...

Interpretations in general

σ-structure $\xrightarrow{I}$ τ-struct.

Definition

Let σ and τ be relational signatures, where $\tau = \{Q_1, \ldots, Q_l\}$. An *interpretation* from σ to τ (or interpretation of τ in σ) is a tuple $I = (\nu, \psi_1, \ldots, \psi_l)$ of FO σ-formulas, where

- ν has one free variable and
- the number of free variables of ψ_i is the same as the arity of Q_i.

An interpretation I from σ to τ defines for every σ-structure $\mathcal{A}$, a τ-structure $\mathcal{B} = (B, Q_i^{\mathcal{B}}, \ldots, Q_i^{\mathcal{B}})$ as follows:

- $B = \{a \in A \mid \mathcal{A} \models \nu(a)\}$
- $Q_i^{\mathcal{B}} = \{(a_1, \ldots, a_k)) \in A^k \mid \mathcal{A} \models \psi_i(a_1, \ldots, a_k)\}$

We denote this by $\mathcal{B} = I(\mathcal{A})$ and extend this to classes: $I(\mathcal{C}) = \{I(\mathcal{A}) \mid \mathcal{A} \in \mathcal{C}\}$.

Example

The class of all trees of height ~~3~~ 2 is interpretable in words over alphabet $\Sigma = \{a, b, c\}$.

$\sigma = \{\leq, P_a, P_b, P_c\}$

binary

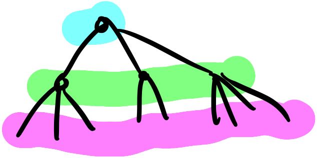
unary

1 2 3 4 5

abbab

$\forall x\, P_a(x) \to \exists y\, (x < y \wedge P_b(y))$

$\tau = \{E\}$

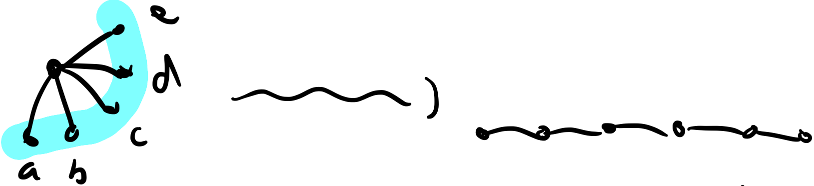


abcccbccbccce

$\varphi(x, y) := (P_a(x) \wedge P_b(y)) \vee (P_b(x) \wedge P_c(y) \wedge$ "x is the closest b to the left of y")

Showing non-interpretability

The class of all paths graphs is not interpretable in the class of stars.



Twin set will form a clique or ind. set after an interpretation

$\psi(a,b)$ ✓

$\psi(a,c)$ ✓

Showing non-interpretability

The class of all grids is not interpretable in any class of bounded clique-width.

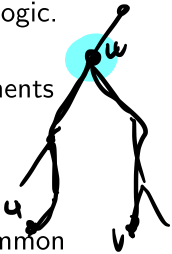
1. If $\mathcal{C}$ is a class of bounded clique-width, then it does not contain all grids
2. If $\mathcal{C}$ is a class of bounded clique-width and I is an interpretation, then $I(\mathcal{C})$ is a class of graphs of bounded clique-width

Part III: Interpretations and sparsity

Some older results

Interpretations in trees have been studied, but for MSO logic.

MSO logic $=$ FO logic $+$ quantification over sets of elements



Can express: connectedness, 3-colorability ,...

$\varphi(x, y) = \dots$

In rooted trees it can express that vertex w is a least common ancestor of vertices u and v – important for interpretations

Results:

- For every k the class of graphs of treewidth at most k is interpretable in colored trees (incidence model)
- For every k the class of graphs of clique-width at most k is interpretable in colored trees (adjacency model)

Sparse graphs

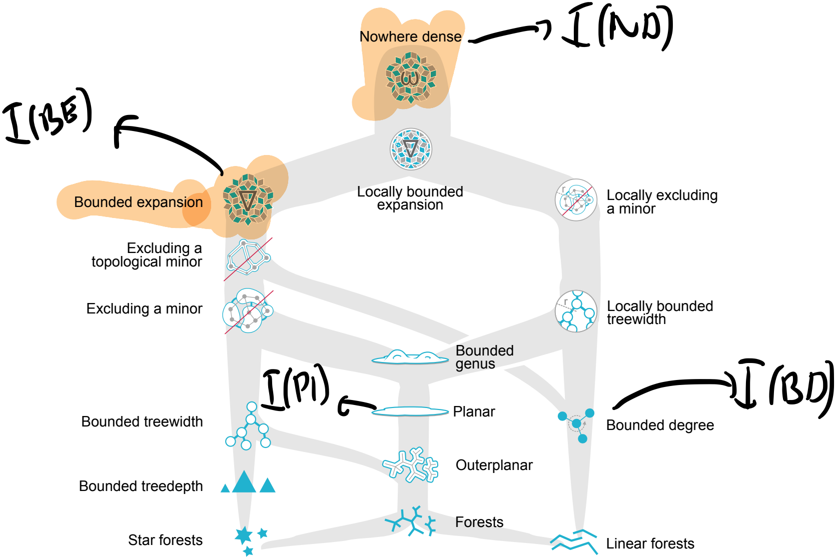


Image by Felix Reidl

Research program

For any sparse graph class $\mathcal{C}$:

- Characterize graph classes interpretable in $\mathcal{C}$
- Find an algorithm to 'reverse' interpretations
- Find a model checking algorithm

INPUT

$$G \overset{\psi}{\rightsquigarrow} H$$

$$\mathcal{C} \qquad \mathcal{D}$$

Research program

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Done for bounded degree and partially for bounded expansion.

Interpretations of bounded expansion graph classes

Bounded expansion graph classes – can be characterized low treedepth colorings/covers

Interpretations of bounded expansion graph classes – can be characterized low shrub-depth colorings/covers

Shrub-depth – dense counterpart of treedepth

Shrub-depth

Fix d (depth) and set C of colors of size m

Fix a relation $R \subseteq C \times C \times \{2, 4, \ldots, 2d\}$

Consider a tree T of depth d such that every leaf has one of m colors

Such a tree is called a *tree-model* and it defines graph G as follows:

- Vertices of G are leaves of T
- There is an edge between u and v if
$$(color(u), color(v), dist(u, v)) \in R$$

Shrub-depth

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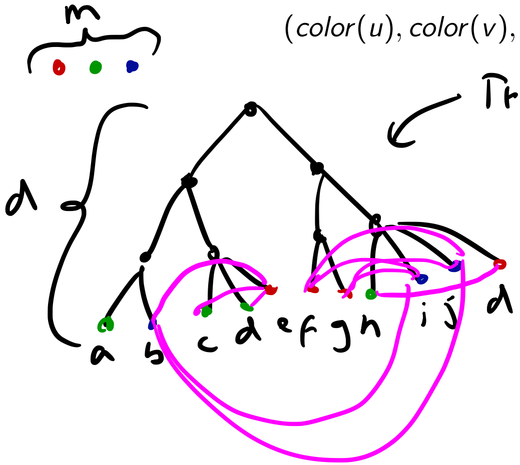
- There is an edge between u and v if

$$(color(u), color(v), dist(u,v)) \in R$$



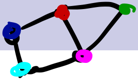
Tree-model

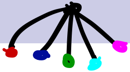
(Red, Green, 2)

(Blue, red, 4)

(Blue, blue, 6)

Shrub depth





Notice that every graph G has a tree model of depth 1 and at most $|V(G)|$ colors

Consequence – it does not make sense to speak about depth of a single graph; depth has to be defined for classes of graphs

Definition

Shrub depth of a class $\mathcal{C}$ of graphs is the least d such that there exists m such that every graph in $\mathcal{C}$ has a tree model of depth d with at most m colors.

Theorem

Class $\mathcal{C}$ of graphs has bounded shrubdepth if and only if it is interpretable in the class of trees of bounded height

Interpretations and sparsity – open problems

What we don't know:

1. Characterization of classes of graphs interpretable in sparse graph classes besides bounded degree and bounded expansion

Interpretations and sparsity – open problems

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Interpretations and sparsity – open problems

What we don't know:

1. Characterization of classes of graphs interpretable in sparse graph classes besides bounded degree and bounded expansion
2. Most importantly – characterization of graph classes interpretable in nowhere dense graph classes
3. Computing reverses of interpretations: Let $\mathcal{C}$ be a class of sparse graphs, I an interpretation and $\mathcal{D} = I(\mathcal{C})$. Given $H \in \mathcal{D}$ as input, compute $G \in \mathcal{C}$ such that $H = I(G)$.

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   - Even an algorithm with runtime $H^{|I|}$ would be interesting

$f(I) \cdot |H|$

$|H|^{f(I)}$

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   - Also using a different interpretation I' would be enough, it is only important that I' is the same for all graphs H

$$\begin{tikzcd} G \arrow[r, "I"] & H \\ G' \arrow[ur, "I'"] & \end{tikzcd}$$

Interpretations and sparsity – open problems

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   - Even an algorithm with runtime $H^{|I|}$ would be interesting
   - Also using a different interpretation I' would be enough, it is only important that I' is the same for all graphs H
   - The computed graph G also does not really have to come from $\mathcal{C}$, it only has to come from a fixed sparse class $\mathcal{C}'$ of graphs

$$\mathcal{C} \xrightarrow{I} \mathcal{D} \xleftarrow{I'} \mathcal{C}'$$

deg 5   deg ≤ 17

Interpretations and sparsity – open problems

The simplest variant of the algorithmic problem from the previous slides:
Let $\mathcal{D}$ be class of graphs interpretable in trees using interpretation I. Find an an interpretation I' and a nowhere class $\mathcal{C}'$ and an algorithm which does the following: Given $H \in \mathcal{C}$ as input, the algorithm finds $G \in \mathcal{C}'$ such that $H = I'(G)$ in time $H^{f(I)}$.