δ-boundedness

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PRINCETON UNIVERSITY

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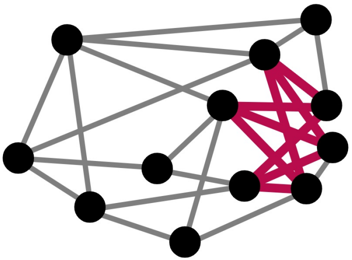
With Xiying Du, António Girão, Zach Hunter, Alex Scott, James Davies, Tomasz Krawczyk, and Bartosz Walczak.

When is $\mathbf{K_{t,t}}$ roughly the densest part of G?



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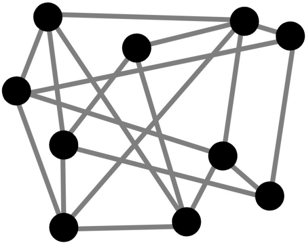

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There exist graphs of arbitrarily large min degree & girth.

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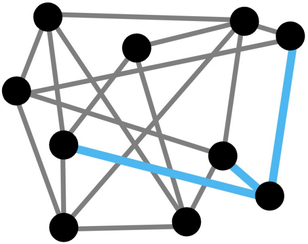

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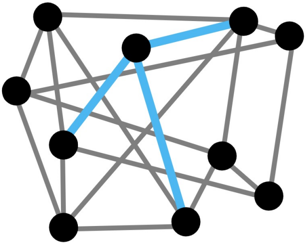

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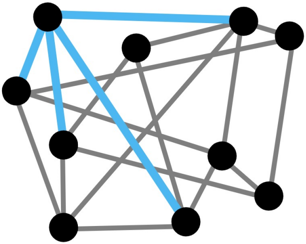

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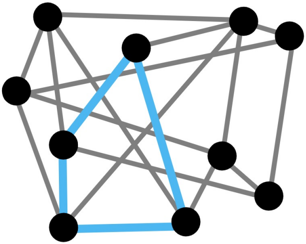

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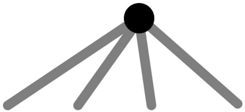

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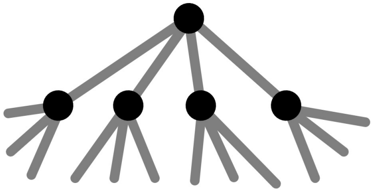

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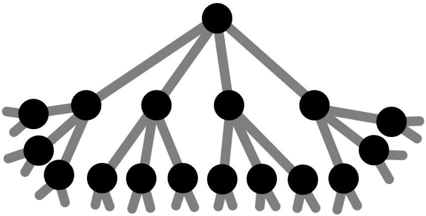

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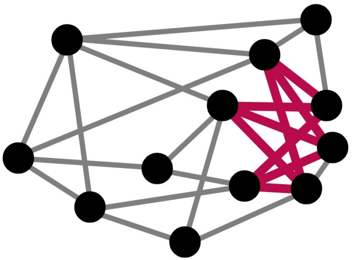

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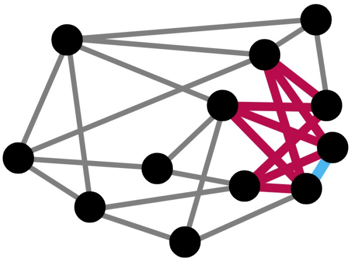

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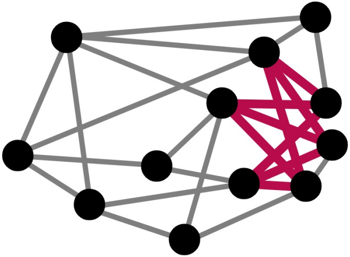

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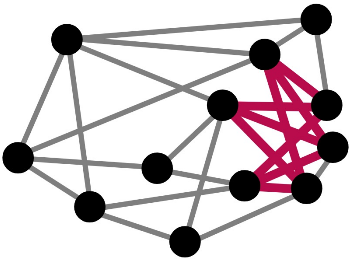

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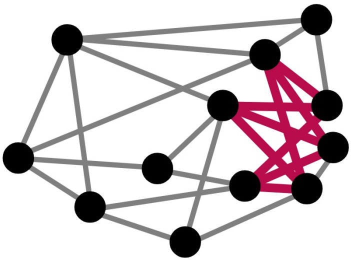

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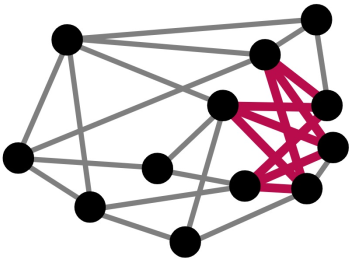

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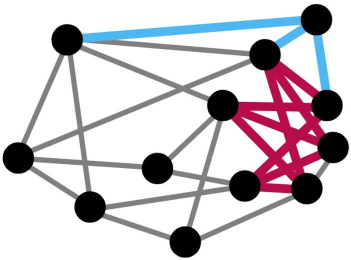

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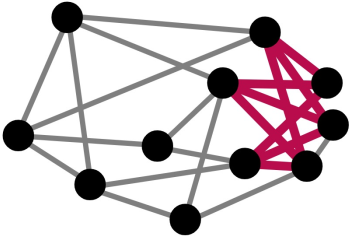

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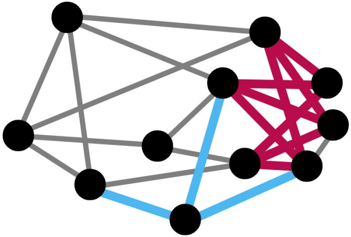

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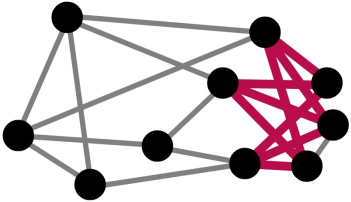

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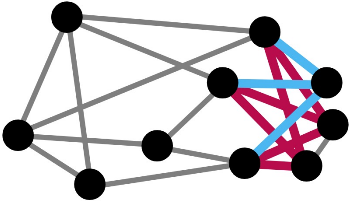

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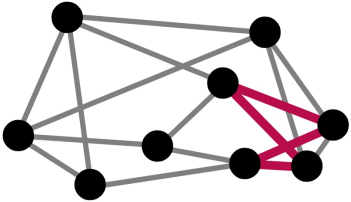

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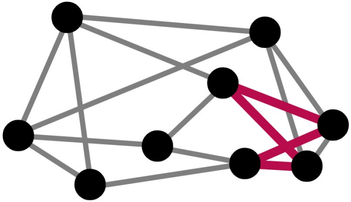

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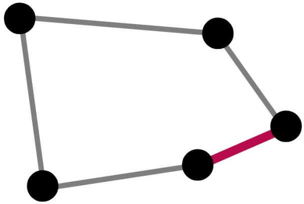

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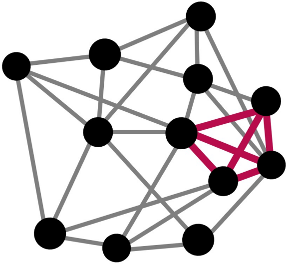

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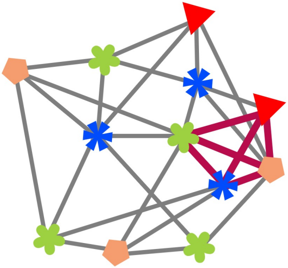
A coloring assigns adjacent vertices different colors.



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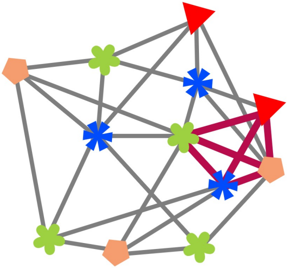

The chromatic num

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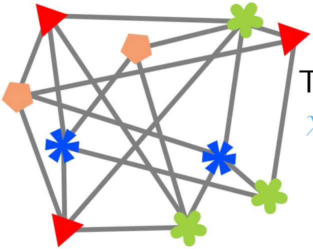

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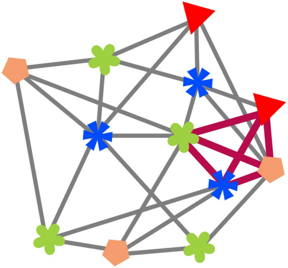

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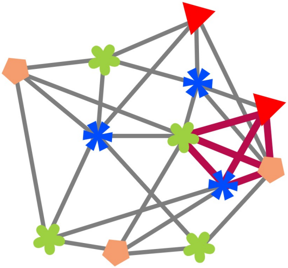

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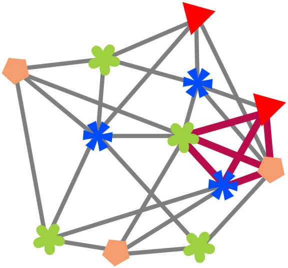

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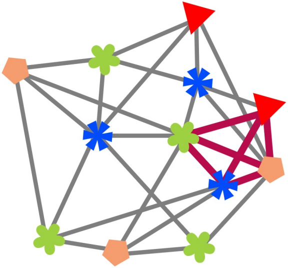

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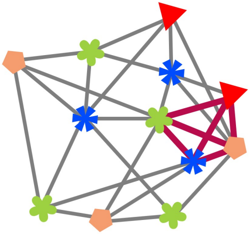

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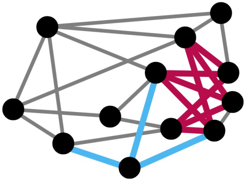

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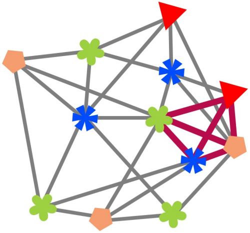

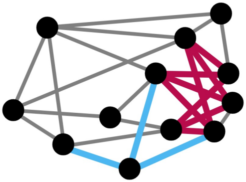

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There exists a class $\mathcal{F}$ which is **not** χ**-bounded** *so that the chromatic number of* $\{H \le_{ind} \mathcal{F} : \omega(\mathbf{H}) = 2\}$ **is** *bounded.*

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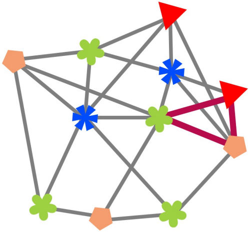

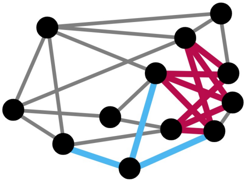

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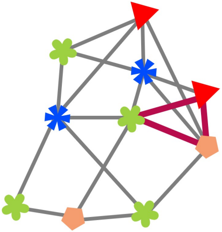

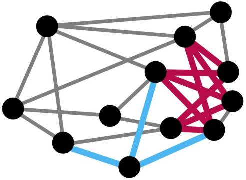

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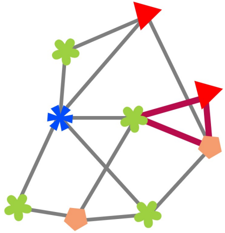

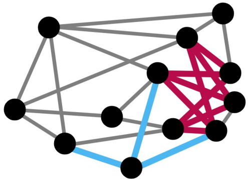

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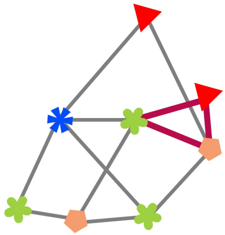

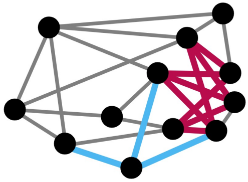

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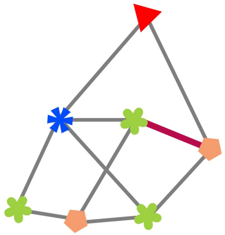

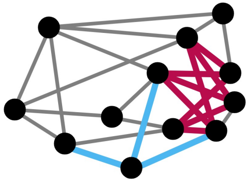

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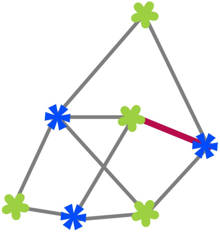

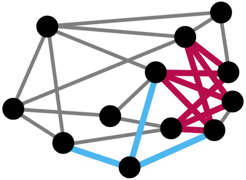

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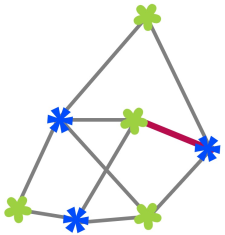

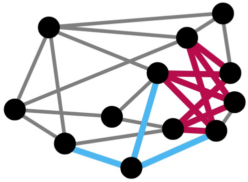

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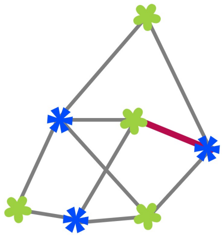

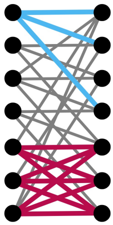

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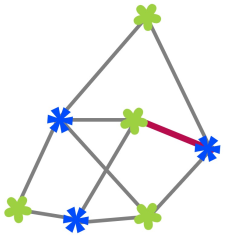

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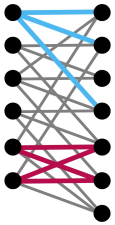

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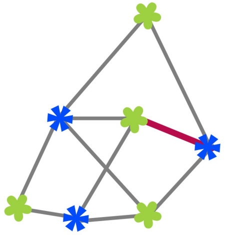

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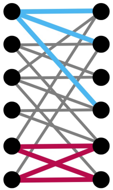

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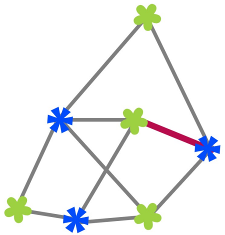

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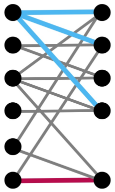

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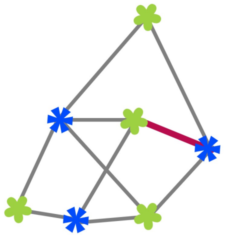

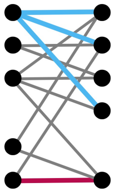

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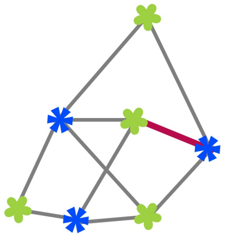

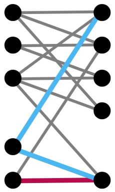

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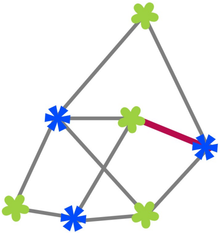

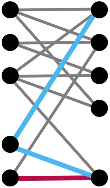

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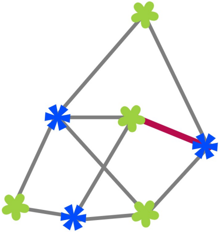

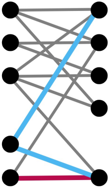

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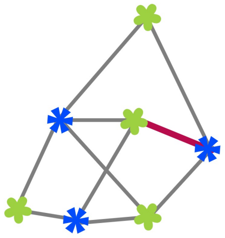

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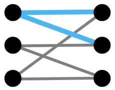

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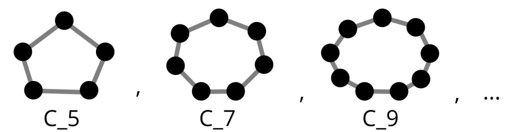

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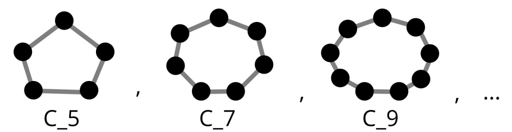

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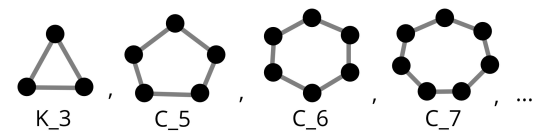

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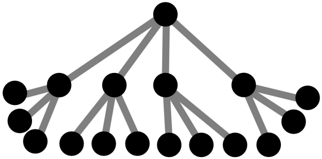

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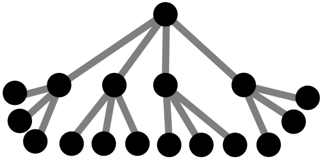

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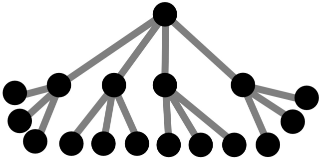

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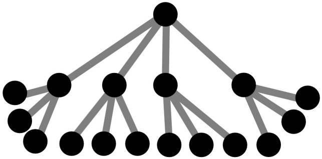

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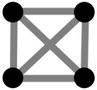

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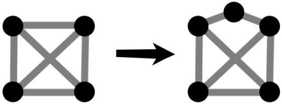

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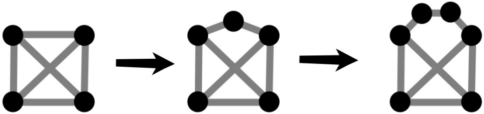

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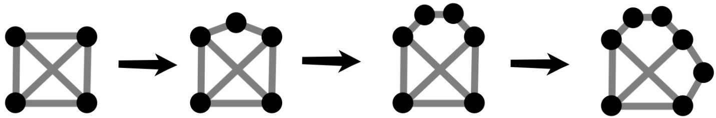

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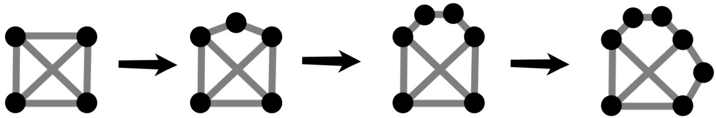

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The class $\mathcal{F}_{K_5^1}$ is **not** χ**-bounded**.

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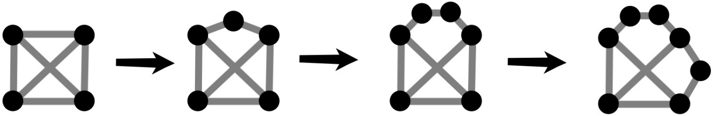

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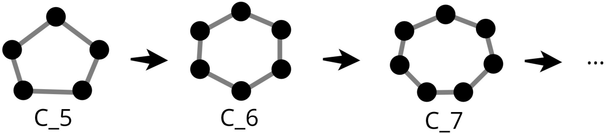

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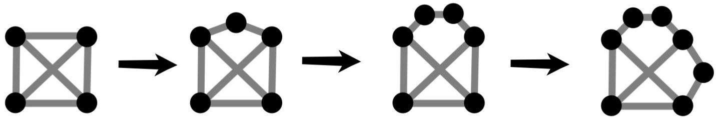

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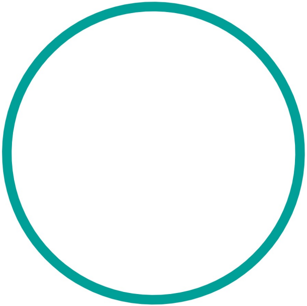
Let $\mathbf{C} \subset \mathbb{R}^2$ be a circle.



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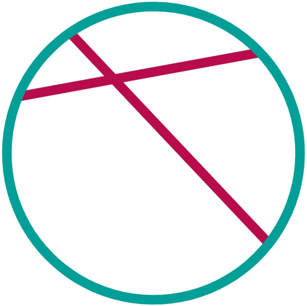

intersecting chords

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Theorem (Gyárfás 85)

For every $\mathcal{R}$, *we have* $\chi \leq 4^{\omega \log(\omega)}$.

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Theorem (Davies-McCarty 21)

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Theorem (Davies 22)

For every $\mathcal{R}$, we have $\chi \leq 15\omega \log(\omega)$.

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Theorem (Fox-Pach 10)

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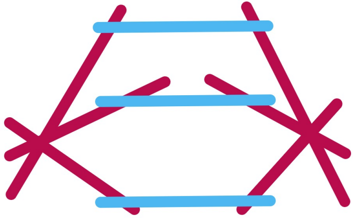

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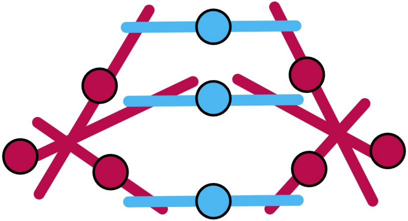

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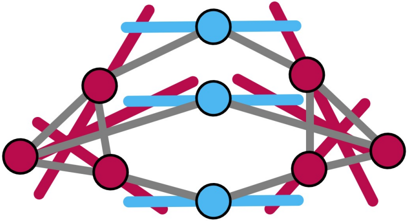

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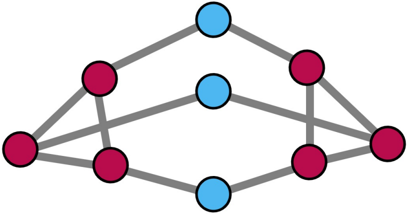

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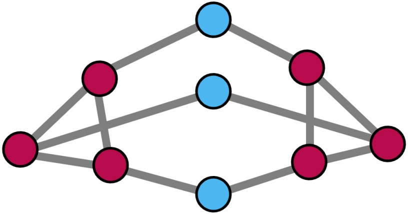

Theorem (Pawlik-Kozik-Krawczyk-Lasoń-Micek-Trotter-Walczak 14)

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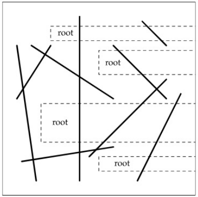

Fig. 1. Segments, probes and roots.

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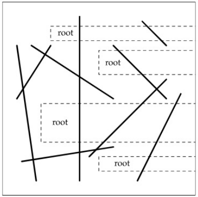

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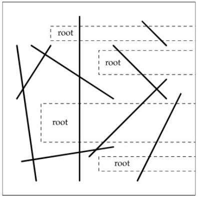

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Can we prove the same for induced-minor-free graphs using recent separator theorem of Korhonen-Lokshtanov?

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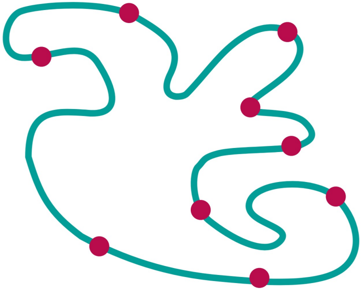

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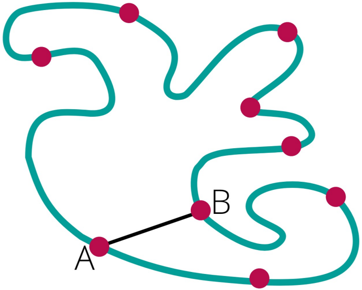

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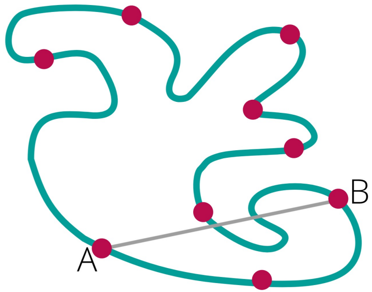

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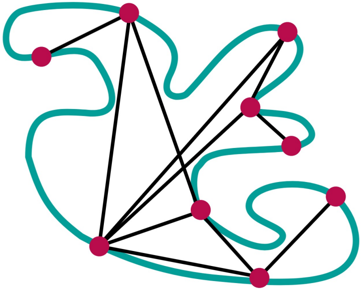

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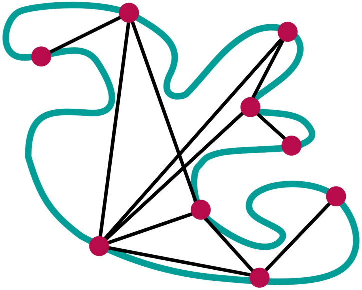

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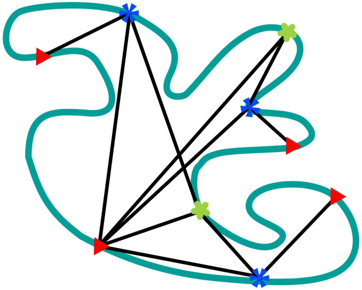

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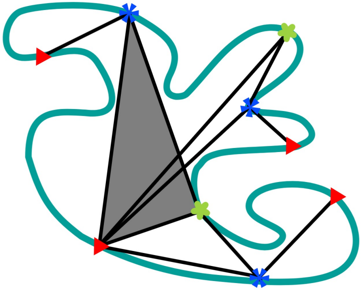

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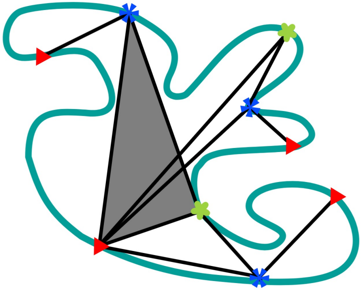

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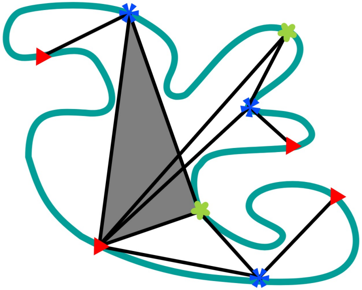

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Question

Is this class **δ-bounded?**

How quickly can optimal bounding functions grow?

$$\chi \leq \omega$$

$$\chi \leq \omega^3$$

$$\chi \leq 2^\omega$$

$$\chi \leq \omega^{\omega^{\omega^{\omega^{\omega^{\omega}}}}}$$

$$\cdots$$

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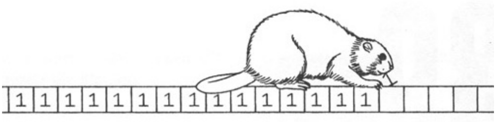

Figure from The New Turing Omnibus, Dewdney

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Some intuition for the bound $\delta \leq 2^{\mathcal{O}(\tau^3)}$

Theorem (Kwan-Letzter-Sudakov-Tran 20)

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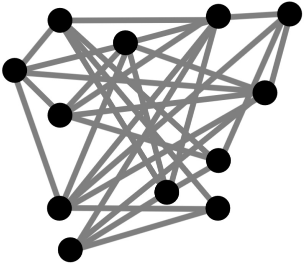

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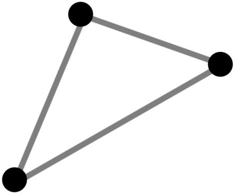

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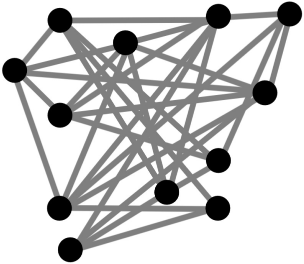

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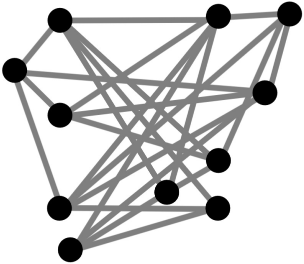

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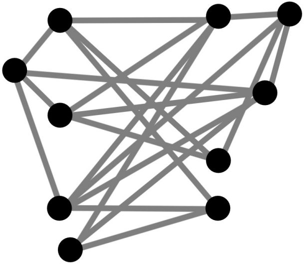

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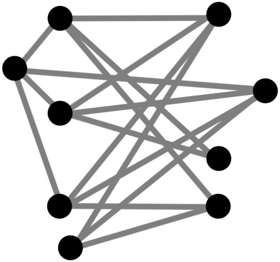

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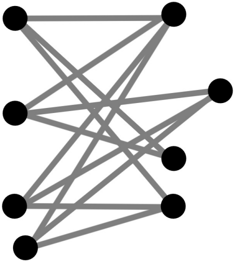

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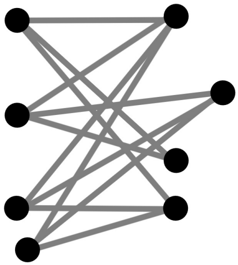
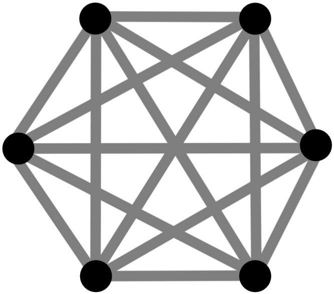


We can do better by assuming there is no "roughly regular" induced subgraph with δ large.

Theorem (Erdös 1959)

There exist graphs of arbitrarily large min degree & girth.



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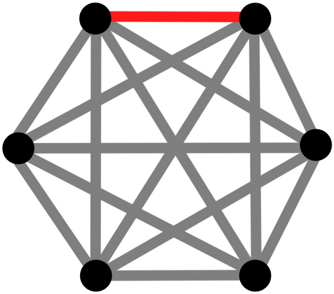

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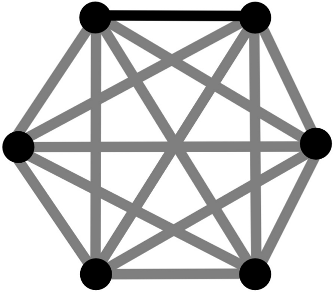

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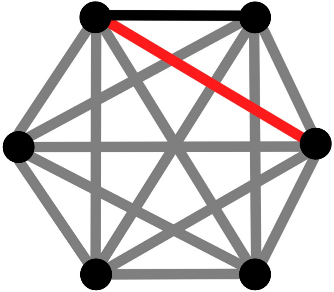

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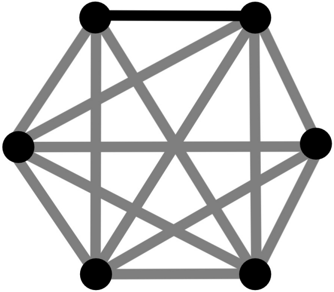

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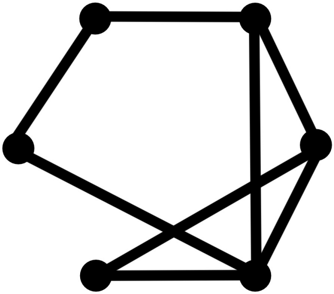
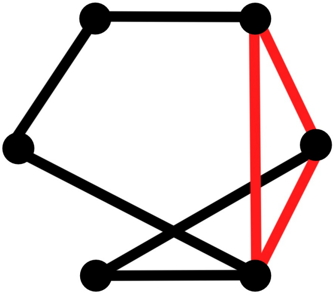


Theorem (Erdös 1959)

There exist graphs of arbitrarily large min degree & girth.



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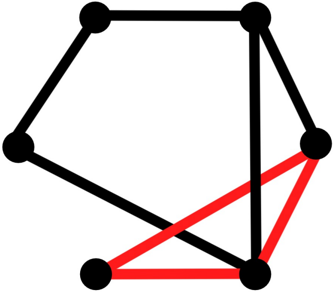

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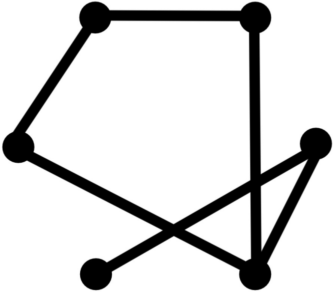
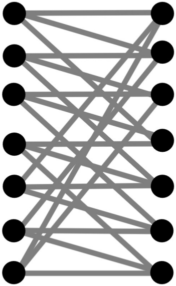


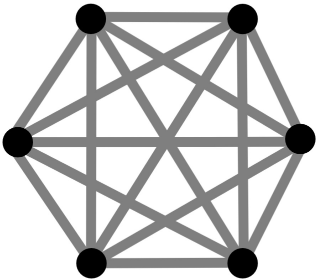
Conjecture (Thomassen 1983)

There exists $f : \mathbb{N} \to \mathbb{N}$ such that for any d, k, ***every*** *graph with $\delta \geq f(d, k)$ has a subgraph with $\delta \geq d$ and girth $\geq k$.*

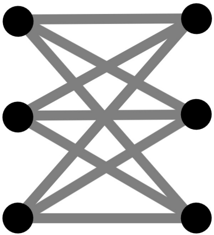


$\delta \geq d$, girth $\geq k$

What if we want an **induced** subgraph of large average degree and girth?



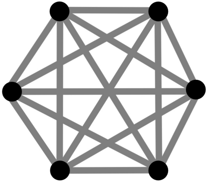
clique K_6



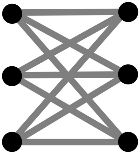
biclique $K_{3,3}$

Conjecture

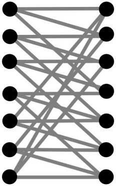
There exists $f : \mathbb{N} \to \mathbb{N}$ such that for any d, k, ***every*** *graph with $\delta \geq f(d, k)$ has as an* **induced** *subgraph either K_d, $K_{d,d}$, or a graph with $\delta \geq d$ and girth $\geq k$.*



clique



biclique



$\delta \geq d$, girth $\geq k$

Conjecture

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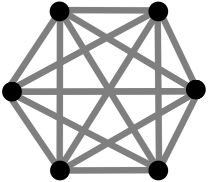

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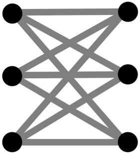

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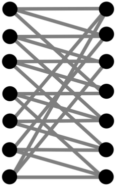

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Implies Thomassen's Conjecture.

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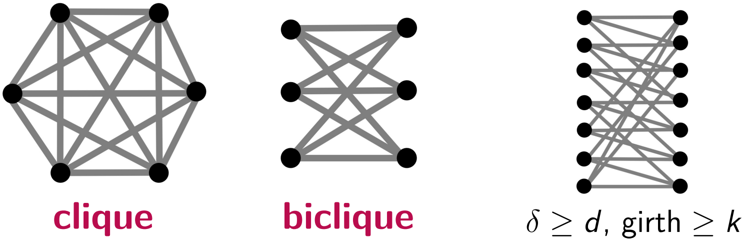
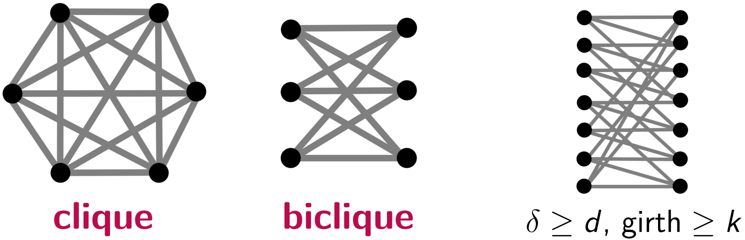


Implies Thomassen's Conjecture.

True for $k = 6$.

Conjecture

There exists $f : \mathbb{N} \to \mathbb{N}$ such that for any d, k, ***every*** *graph with $\delta \geq f(d,k)$ has as an* **induced** *subgraph either K_d, $K_{d,d}$, or a graph with $\delta \geq d$ and girth $\geq k$.*



Implies Thomassen's Conjecture.

True for $k = 6$.

True for "roughly regular graphs".

Thank you!