Discrete Mathematics

Exercise sheet 8

21/ 27 November 2016

1. Last week we read the sorry tale of the restaurant cook who lost both her engagement ring and her wedding ring in a large pot of soup, all of which was served up evenly among 25 guests. In a development of the plot, upon receiving his plate of soup one of the guests huffily declared it to be a "dog's dinner" and thereupon gave it to the restaurant dog sitting hungrily in the corner. What is the probability that

   (a) the dog finds the wedding ring in its soup, given that it also finds the engagement ring?

   (b) the dog finds both rings in its soup, given that it finds at least one?

2. In a random string of one hundred bits, in which each bit takes value 0 or 1 with equal probability, and different bits are independent:

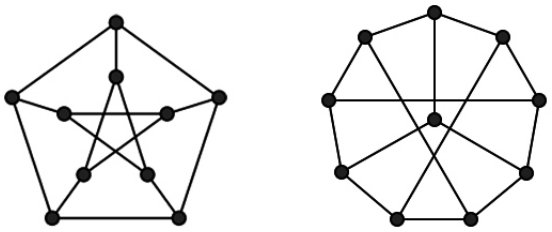
   (a) Determine the probability that 6 consecutive bits of the string are all equal to 1.

   (b) Find the expected number of 6 consecutive 1s in the string (i.e., number of substrings 111111). [*Hint: introduce indicator variables for an occurrence of substring* 111111 *beginning at position* i *(*$1 \leq i \leq 95$*) and use linearity of expectation.*]

   (c) More generally, for $0 \leq k \leq n$, in a random string of n independent bits, the values 0 and 1 occurring with equal probability in each position, determine the expected number of k consecutive 1s.

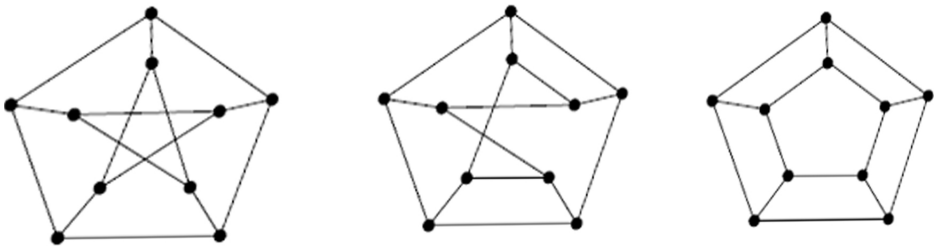
   (d) A *switch* occurs when consecutive bits are different. Determine the expected number of switches in a random string of n bits.

3. Find an isomorphism between the following graphs:



   What length cycles do these graphs have?

4. Explain why no two of the following graphs are isomorphic:



   What property do the three graphs have in common (other than each having 10 vertices)?