Discrete Mathematics

Exercise sheet 7

14/ 20 November 2016

1. A restaurant cook had the misfortune of losing her engagement ring in a big cauldron of soup, and the carelessness to lose her wedding ring after it, which also found its way into the soup. The cook was for some reason not too painstaking in searching for her lost rings and served up all the soup until the pot was completely empty. The soup was divided among 25 guests, among whom 8 were women. What is the probability that

   (a) one person got both rings?

   (b) no man got a ring?

   (c) two men got a ring each?

   (d) a man got one ring, a woman the other?

2. In this question we assume dice are fair (unbiased), in the sense that each of the six possible scores of a die are equally likely. Additionally, successive throws of a die are independent, the outcome of one throw not affecting that of another.

   (a) What is the probability that throwing two dice yields an even total score? A multiple of 3?

   (b) Determine the probability six throws of a die yield a score of three or more at least three times.

   (c) What is the probability that in three successive throws the scores are strictly increasing?

   (d) What is the probability that in three successive throws the scores are nondecreasing?

   (e) Answer (c) and (d) for k successive throws rather than three, where $k \in \mathbb{N}$.

3. (Birthday Paradox) For this problem we ignore leap years and assume that a person has his/her birthday among one of the 365 days of the calendar year.

   (a) Show that there is more than a 50% chance that in a room containing 23 people there are two who share a birthday. [*Hint: show that the probability that everyone has different birthdays is less than $\frac{1}{2}$, using a calculator/computer to perform the arithmetic.*]

   (b) Use the approximation $1 - x \approx e^{-x}$ for small x to show that among n people the chance that two people share a birthday is close to $1 - e^{-n^2/730}$.