Discrete Mathematics

Exercise sheet 2

10 /13 October 2016

1. Which of the following functions from $\mathbb{Z}$ to $\mathbb{Z}$ are injections, and which of them are bijections?

   (a) $f(x) = x^2$

   (b) $g(x) = x - 3$

   (c) $h(x) = 3x + 1$

   (d) $i(x) = x^2 - 1$

2. Let $f : X \to Y$ and $g : Y \to Z$ be functions such that $(g \circ f)(x) = x$ for each $x \in X$ and $(f \circ g)(y) = y$ for each $y \in Y$. Prove that f and g are bijections.

3.

   (a) Let A be a set. What is the set $A \times \emptyset$ equal to?

   (b) Let A, B, C be sets. Under what conditions does it follow from $A \times C = B \times C$ that $A = B$?

4. Let X be a finite set and let 2^X denote the set of all subsets of X.

   (a) Prove that $|2^X| = 2^{|X|}$.

   (b) Prove that $2^X = 2^Y$ if and only if $X = Y$.

5. Describe the relation $R \circ R$ if R stands for

   (a) the equality relation "=" on the set $\mathbb{N}$ of natural numbers,

   (b) the relation "less than or equal to" ("$\leq$") on $\mathbb{N}$,

   (c) the relation "strictly less than" ("$<$") on $\mathbb{N}$,

   (d) the relation "strictly less than" ("$<$") on the set $\mathbb{R}$ of real numbers.