INTRO TO APPROXIMATION – HW2

Deadline: Tuesday **December 19, 2017 23:59 AoE** (AoE means Anywhere on Earth; if you send your solutions after midnight of Central European Time, mention a country in which it is still December 19).

Send your solutions preferably by email (in PDF, ODT ...or just inside the email). I also accept scans or photos of quality high-enough to read everything without problems. Another possibility is to hand your solutions in on a paper at the beginning of a class. Please ask if any homework is not clear to you or if you think that the description below is missing something.

HOMEWORK ONE — **MAXIMUM k-CUT** — [5 points]

On the input for MAXIMUM k-CUT we get an undirected graph G and weights on the edges $w\colon E(G) \to \mathbb{R}^+$. Our goal is to partition the vertices into k disjoint sets $V_1, V_2, \ldots, V_k$ so that we **maximize** the sum of weights of all "multicolored" edges (those that go from any one set to any other set).

Suggest and analyze a $\frac{k-1}{k}$-approximation algorithm for MAXIMUM k-CUT. A deterministic algorithm is preferred, but for a randomized algorithm you get 3 points.

HOMEWORK TWO — **SONET RING-LOADING PROBLEM** — [5 points]

Design a 2-approximation algorithm for SONET RING-LOADING PROBLEM.

In this problem we have a network – a cycle with n vertices. On input we get a list of requests, each request having a source vertex and a target vertex. Every request needs to be assigned one out of two possible paths from source to target (either clockwise or counterclockwise).

Our goal is to minimize the load (total number of uses) of the most loaded edge of the cycle.

TŘETÍ DOMÁCÍ ÚKOL — **Parallel sorting with many processors** — [5 points]

Sort n numbers which are in memory (each in one cell) using n^2 processors in time $\mathcal{O}(\log n)$. More processors can read one cell concurrently, but only one processor can write to a cell in one step.

Hint: you have enough processors to compare each pair of numbers. Randomization is not needed, there is a deterministic algorithm.

(There is a randomized algorithm for sorting n numbers in $\mathcal{O}(\log n)$ using n processors.)

ČTVRTÝ DOMÁCÍ ÚKOL — **Positive SAT** — [6 points]

In this variant of maximum satisfiability problem (with boolean formulas in CNF) variables occur only positively in clauses (there is no negation). Moreover, each clause and each variable has a nonnegative weight. The goal is to maximize the total weight of satisfied clauses plus the total weight of variables set to *false*.

Use the relaxation of the integer program for MAX SAT and a randomized rounding in which we set the i-th variable of the formula to *true* with probability $1 - \lambda + \lambda y_i^*$, where $\lambda = 2(\sqrt{2} - 1) \approx 0.828$ and y_i^* is the optimal value for the i-th variable of the formula (more precisely, y^*, z^* is the optimal solution of the relaxation in which y are variables for variables of the formula and z are variables for clauses). Show that such a rounding is a λ-approximation algorithm.

Hint: For any natural $k \geq 1$ you can use as a fact that the function $f(x) = 1 - \lambda^k \left(1 - \frac{x}{k}\right)^k$ is concave, and also the following inequality (tight for $k = 2$ — this case determines the optimal value of λ)

$$1 - \lambda^k \left(1 - \frac{1}{k}\right)^k \geq \lambda\,.$$