Chernoff bounds

Notations

$X_i \in [0,1]$, $i = 1, \ldots, n$ – **independent** random variables

$X = \sum_{i=1}^{n} X_i$

$\mu = E[X] = \sum_{i=1}^{n} E[X_i]$

$\delta \in (0,1)$

Multiplicative error bounds (depending on the mean)

$$Pr[X \leq (1-\delta)\mu] \leq \left(\frac{e^{-\delta}}{(1-\delta)^{(1-\delta)}}\right)^{\mu} \leq e^{-\frac{\delta^2\mu}{2}}$$

$$Pr[X \geq (1+\delta)\mu] \leq \left(\frac{e^{\delta}}{(1+\delta)^{(1+\delta)}}\right)^{\mu} \leq e^{-\frac{\delta^2\mu}{2+\delta}} \leq e^{-\frac{\delta^2\mu}{3}}$$

$$Pr[X \geq R] \leq 2^{-R}, \qquad R \geq 6\mu$$

Additive error bounds (symmetric)

$$Pr[X \geq \mu + \delta\sqrt{n}], \quad Pr[X \leq \mu - \delta\sqrt{n}] \quad \leq \quad e^{-2\delta^2}$$

$$Pr[X \geq \mu + \delta n], \quad Pr[X \leq \mu - \delta n] \quad \leq \quad e^{-2n\delta^2}$$

Binomial distribution

$X_i \in \{0,1\}$, $i = 1, \ldots, n$ – **independent uniformly random** variables

$$Pr[X \geq (1+\delta)\mu], \quad Pr[X \leq (1-\delta)\mu] \quad \leq \quad e^{-\delta^2\mu}$$