NDMI025 – Randomized Algorithms (Pravděpodobnostní algoritmy) LS 2015 – Jiří Sgall

Homework 4 – May 3

Due date: May 18 before the lecture

Recitation session for this homework set will take place on May 21

Sign all the sheets either by your name or by your nickname.

Every exercise is worth 2 points.

(1) In the lecture we have used randomized sampling to approximately count. Now we shall do the converse.

Assume you have a FPRAS (fully polynomial randomized approximation scheme) to approximate the number of perfect matchings in a graph on input. Design a polynomial time randomized algorithm to approximately uniformly sample a perfect matching in a given graph. (If you find it convenient, you may assume that the graphs are bipartite and dense.) Find an estimate on the quality of your algorithm, i.e., the distance of its distribution to the uniform one.

(2) Consider a random walk on a hypercube of dimension n which moves to each of n neighbors of a given node with probability $1/n$. Using coupling, prove that after $O(n \ln(n/\varepsilon))$ steps from any initial distribution, the distribution is ε-close to uniform.

(3) Let T be a complete ternary tree of depth d with majority gates at the inner nodes and 3^d leaves containing a Boolean assignment. Assume that we assign each leaf independently 1 with probability $1/2 + x$ and 0 with probability $1/2 - x$ for some $x > 0$. Estimate the probability that the root of the tree evaluates to 1.

Hint: It is easy to set up a recurrence. Then it is needed to analyze separately the regions where the probability is close to 1/2 and close to 1.

(4) Prove that there exists a polynomial size monotone formula for computing majority of n Boolean variables. (Monotone formula may use AND and OR gates, no negations.)

Hint: Consider the ternary tree of majorities for a suitable $d = \Theta(\log n)$, and put independently a random variable at each leaf. Use the previous exercise.