Flows and cycles in graphs – Exercises 2

1. Prove that a cubic graph with a bridge has no edge-3-coloring, without using NZ flows.

2. When proving that the Petersen graph does *not* have some property (in the previous set of exercises we discussed edge 3-coloring, resp. NZ $\mathbb{Z}_2^2$-flow) it is helpful, that the graph is extremely symmetric. Proving these symmetries is the topic of this exercise. First few ad-hoc definitions:

We say that graph G is *H-transitive*, if whenever H_1, H_2 are subgraphs of G, both isomorphic to H, there is an automorphism of G which maps H_1 to H_2.

We say that graph G is *ordered H-transitive*, if whenever H_1, H_2 are subgraphs of G, both isomorphic to H, and $f : H_1 \to H_2$ is an isomorphism, then there is an automorphism of G which extends f.

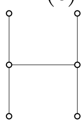
(a) *Kneser graph* $K(n, k)$ is a graph which has k-subsets of an n-set as vertices, and two vertices are adjacent iff the corresponding sets are disjoint. Show that the Petersen graph is isomorphic with $K(5, 2)$.

(b) The Petersen graph is K_1-transitive (or vertex-transitive).

(c) The Petersen graph is K_2-transitive (or edge-transitive).

(d) The Petersen graph is ordered K_2-transitive (or arc-transitive).

(e) The Petersen graph is ordered H-transitive where H is a tree in the figure.



(f) The Petersen graph is M-transitive, where M is a matching with 5 edges.

3. Prove that the Flower snark (in the figure) is not 3-edge-colorable (so it is indeed a snark).