(1) $\displaystyle\int_0^1 x^2 \arcsin x =$

$\left(\int_0^1 f\,G = [FG]_0^1 - \int_0^1 F g\right)$

$$= \left[\frac{x^3}{3}\arcsin x\right]_0^1 - \int_0^1 \frac{x^3}{3}\,\frac{1}{\sqrt{1-x^2}}$$

$\left[\frac{x^3}{3}\arcsin x\right]_0^1 = \frac{\pi}{6}$

$\int_0^1 \frac{x^3}{3}\frac{1}{\sqrt{1-x^2}} =: I$ subst. $y = 1-x^2$ $\quad \varphi'(x) = -2x \neq 0$

$dy = -2x\,dx$

$$I = \int_1^0 \frac{1-y}{3}\cdot\frac{1}{\sqrt{y}}\,\frac{dy}{-2}$$

$$= \frac{1}{6}\int_0^1 \left(y^{-\frac{1}{2}} - y^{\frac{1}{2}}\right) dy$$

$$= \frac{1}{6}\left[2y^{\frac{1}{2}} - \frac{2}{3}y^{\frac{3}{2}}\right]_0^1 = \frac{1}{6}\left(2-\frac{2}{3}\right) = \frac{2}{9}$$

výsledek: $\boxed{\frac{\pi}{6} - \frac{2}{9}}$

② $f(x)$ je spojitá na $\mathbb{R}$ $\Rightarrow$ má na celém $\mathbb{R}$ prim. funkci $F(x)$.

Použijeme subst. $t = \operatorname{tg} x$, pak budeme lepší.

$$f(x) = \frac{1}{\sin^2 x + 25\cos^2 x + 2} = \frac{\frac{1}{\cos^2 x}}{\operatorname{tg}^2 x + 25 + \frac{2}{\cos^2 x}}$$

$$= \frac{\frac{1}{\cos^2 x}}{t^2 + 25 + 2(t^2+1)} = \frac{\frac{1}{\cos^2 x}}{3(t^2+9)}$$

$$\int f(x) = \left| \begin{array}{c} t = \operatorname{tg} x \\ \cos^2 x = \frac{1}{1+t^2} \\ dt = \frac{1}{\cos^2 x}\, dx \end{array} \right| = \int \frac{dt}{3(t^2+9)}$$

$$= \frac{1}{\cancel{3}9} \int \frac{1}{\left(\frac{t}{3}\right)^2 + 1} \frac{dt}{3} = \frac{1}{\cancel{3}9} \operatorname{arctg} \frac{t}{3} + C$$

$$= \underbrace{\frac{1}{9} \operatorname{arctg} \frac{\operatorname{tg}(x)}{3}}_{F_0(x)} + C$$

Abychom našli p-fci na celém $\left(-\frac{\pi}{2}, \frac{3\pi}{2}\right)$, musíme fci „slepit" v $x = \frac{\pi}{2}$" :

$$F(x) = \begin{cases} F_0(x) & x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \\ F_0(x) + C_0 & x \in \left(\frac{\pi}{2}, \frac{3\pi}{2}\right) \end{cases}$$

$$\left.\begin{aligned} F_0\left(\tfrac{\pi}{2}_-\right) &= \frac{\pi}{\cancel{4}18} \\ F_0\left(\tfrac{\pi}{2}_+\right) &= -\frac{\pi}{\cancel{6}18} \end{aligned}\right\} \Rightarrow C_0 = \frac{\pi}{\cancel{3}9}$$

Všechny p-fce na $\left(-\frac{\pi}{2}, \frac{3\pi}{2}\right)$ jsou $F(x) + C$.

(5) $f(x) = a + b\sqrt{x}$ $\qquad a=1 \quad b=6 \quad c=3$

$$\text{plocha} = \int_0^c 2\pi f(x) \sqrt{1+f'(x)^2}$$

$$= \int_0^c 2\pi (a + b\sqrt{x}) \sqrt{1+\left(\frac{b}{2\sqrt{x}}\right)^2}\, dx$$

$$= \underbrace{\int_0^c 2\pi a \sqrt{1+\frac{b^2}{4x}}\, dx}_{I_1} + \underbrace{\int_0^c 2\pi b \sqrt{x+\frac{b^2}{4}}\, dx}_{I_2}$$

$$I_2 = \left[2\pi b \frac{2}{3}\Big(x+\underbrace{\frac{b^2}{4}}_{9}\Big)^{\frac{3}{2}}\right]_0^{c\,3} = 6\cdot\frac{4\pi}{3}\Big[\underbrace{12^{3/2}}_{8.3\sqrt{3}} - \underbrace{9^{3/2}}_{27}\Big] = \pi\left(64.3\sqrt{3} - 8.27\right)$$

$$\sqrt{1+\frac{b^2}{4c}} = 2$$

$$I_1 = \int \text{subst.} \left(\begin{aligned} t &= \sqrt{1+\frac{b^2}{4x}} \\ x &= \frac{b^2}{4}\frac{1}{t^2-1} \\ dx &= \frac{b^2}{4}\frac{-1}{(t^2-1)^2}2t \end{aligned} \right) = \int_\infty^{2} 2\pi a\, t\cdot\frac{b^2}{4}\frac{-2t}{(t^2-1)^2}\, dt$$

$$-k = \frac{2\pi}{4}\cdot(-2)\cdot 6^2 = 36\pi$$

$$= k\int_L^\infty \frac{t^2}{(t^2-1)^2}$$

$L = 2$ $\qquad k = 36\pi$

$$\frac{t^2}{(t-1)^2(t+1)^2} = \frac{A}{(t-1)^2} + \frac{B}{t-1} + \frac{C}{(t+1)^2} + \frac{D}{t+1}$$

$$t^2 = A(t+1)^2 + B(t-1)(t+1)^2 + C(t-1)^2 + D(t+1)(t-1)^2$$

$t=1$ $\quad 1 = A\cdot 4 \implies A = \frac{1}{4}$

$t=-1$ $\quad 1 = C\cdot 4 \implies C = \frac{1}{4}$

$t=0$ $\quad 0 = A - B + C + D \implies B - D = \frac{1}{2}$

$t=\infty$ $\quad 0 = B + D$ $\quad \implies B = \frac{1}{4}$, $D = -\frac{1}{4}$

$$\int_2^\infty \frac{t^2}{(t^2-1)^2} = \left[\frac{1}{4}\,\frac{-1}{t-1} + \frac{1}{4}\ln(t-1) + \frac{1}{4}\,\frac{-1}{t+1} - \frac{1}{4}\ln(t+1)\right]_2^\infty$$

$$= \frac{1}{4}\left[-\frac{1}{t-1} - \frac{1}{t+1} + \ln\frac{t-1}{t+1}\right]_2^\infty$$

$$= \frac{1}{4}\left(\frac{1}{2-1} + \frac{1}{2+1} - \ln\frac{2-1}{2+1}\right)$$

$$= \frac{1}{4}\left(1 + \frac{1}{3} - \ln\frac{1}{3}\right) = \frac{1}{3} + \frac{1}{4}\ln 3$$

---

výsledek $\quad \pi\left(3\cdot 64\cdot\sqrt{3} - 8\cdot 27 + \left(\frac{1}{3} + \frac{1}{4}\ln 3\right)36\right)$

$32\pi\sqrt{3}$

-36π

$$= \boxed{3\pi\left(64\sqrt{3} + 3\ln 3 - 68\right)}$$