## 2. konf. pís.

① Napočítejte [illegible]

$$\lim_{x\to\infty} \frac{\operatorname{arctg}\frac{x^3}{1+x} - \frac{\pi}{2}}{\frac{1}{x^2}} \quad \text{— typ } \frac{0}{0}$$

$\hookrightarrow \lim \frac{1}{x^2} = 0$ ✓

$\lim \operatorname{arctg} \frac{x^3}{1+x} = \frac{\pi}{2}$

neboť $\lim\limits_{x\to\infty} \frac{x^3}{1+x} = \infty$

$= \lim\limits_{y\to\infty} \operatorname{arctg} y = \frac{\pi}{2}$

l'Hospital

$$\Rightarrow \; = \lim \frac{\frac{1}{1+\left(\frac{x^3}{1+x}\right)^2} \cdot \frac{3x^2(1+x) - x^3\cdot 1}{(1+x)^2}}{\frac{-2}{x^3}}$$

$$= \lim \frac{(3x^2 + 2x^3)\cdot x^3}{(1+x)^2 + x^6} \cdot \frac{1}{-2}$$

$$= \lim \frac{\left(\frac{3}{x} + 2\right)}{\frac{(1+x)^2}{x^6} + 1} \cdot \frac{1}{-2} = \boxed{-1}$$

(2) Řada $\sum n^{\alpha}\left(1-e^{\frac{1}{(n+1)^2}}\right)$ má vš. členy $<0$,

zkoumejme řadu $\sum n^{\alpha}\left(e^{\frac{1}{(n+1)^2}}-1\right)$

srovnáním s $\sum n^{\alpha-2}$:

$$\lim_{n\to\infty} \frac{n^{\alpha}\left(e^{\frac{1}{(n+1)^2}}-1\right)}{n^{\alpha-2}}$$

$$\lim_{n\to\infty} \frac{e^{\frac{1}{(n+1)^2}}-1}{\frac{1}{(n+1)^2}}\cdot\frac{n^2}{(n+1)^2}$$

$$\lim_{n\to\infty} \frac{e^{\frac{1}{(n+1)^2}}-1}{\frac{1}{(n+1)^2}} \quad \lim \frac{n^2}{(n+1)^2} \to 1$$

$$= \lim_{x\to 0} \frac{e^{x}-1}{x}$$ MLPSS (Heineho věta, $x=\frac{1}{(n+1)^2}$)

$= 1$ (defin. exp)

$0<1<\infty \Rightarrow$ obě řady „konvergují stejně"

$\sum n^{\alpha-2}$ konv. $\Leftrightarrow \alpha-2<-1 \rightarrow \alpha<1$

Závěr: Řada konv. (absolutně) $\Leftrightarrow \alpha<1$, jinak diverguje.

(3) $f(x) = |x+1|^{\frac{1}{3}} - |x-1|^{\frac{1}{3}}$

všimněme si, že $(|x|)' = \operatorname{sgn} x = \begin{cases} 1 & x > 0 \\ -1 & x < 0 \\ \text{nedf} & x = 0 \end{cases}$

(většinou def sgn $0 = 0$, ale to se nám nehodí).

Podle std. metod pro $x \neq \pm 1$ je

$$f'(x) = \frac{1}{3}|x+1|^{-\frac{2}{3}} \cdot \operatorname{sgn}(x+1) - \frac{1}{3}|x-1|^{-\frac{2}{3}} \operatorname{sgn}(x-1)$$

$$f'_+(1) = \lim_{x \to 1_+} f'(x) = \lim_{x \to 1_+} \frac{1}{3}\underbrace{(x+1)^{-\frac{2}{3}}}_{\to 2^{-2/3}} \cdot 1 - \frac{1}{3}\underbrace{|x-1|^{-\frac{2}{3}}}_{\to \infty} \cdot 1 = -\infty$$

$$f'_-(1) = \lim_{x \to 1_-} f'(x) = \lim_{x \to 1_-} \frac{1}{3}(x+1)^{-\frac{2}{3}} \cdot 1 - \frac{1}{3}|x-1|^{-\frac{2}{3}} \cdot (-1) = +\infty$$

$$f'_+(-1) = \lim_{x \to -1_+} f'(x) = \lim_{x \to -1_+} \frac{1}{3}\underbrace{(x+1)^{-\frac{2}{3}}}_{\to \infty} \cdot 1 - \frac{1}{3}\underbrace{|x-1|^{-\frac{2}{3}}}_{\to 2^{-2/3}} \cdot 1 = +\infty$$

$$f'_-(-1) = \lim_{x \to -1_-} f'(x) = \lim_{x \to -1_-} \frac{1}{3}(x+1)^{-\frac{2}{3}}(-1) - \frac{1}{3}|x-1|^{-\frac{2}{3}} \cdot 1 = -\infty$$

Pro $x = \pm 1$ neex. $f'(x)$, neboť $f'_+(x) \neq f'_-(x)$.