NTIN 100 Intro to Info Transmission and Processing summer 2016/2017

5th homework assignment - Communication complexity

two days before the exam

Problem 1. Let $LESS(x, y) : \{0, 1\}^n \times \{0, 1\}^n \to \{0, 1\}$ be a function that is 1 if and only if $\sum_{i=1}^{n} x_i \cdot 2^i < \sum_{i=1}^{n} y_i \cdot 2^i$.

a) Show that its deterministic communication complexity satisfies $D(LESS) \leq n + 1$.

b) Show that its deterministic communication complexity satisfies $D(LESS) \in \Omega(n)$.

c) Show that its non-deterministic communication complexity satisfies $N^0(LESS) \in \Omega(n)$.

d) Show that its non-deterministic communication complexity satisfies $N^1(LESS) \in \Omega(n)$.

e) Show that its randomized communication complexity satisfies $R_{1/4}(LESS) \in O(\log^2 n)$.

Problem 2. Let $DISJ(x, y) : \{0, 1\}^n \times \{0, 1\}^n \to \{0, 1\}$ be a function that is 1 if and only if there exists $i \in \{1, \ldots, n\}$ such that $x_i = y_i = 1$.

a) Show that its non-deterministic communication complexity satisfies $N^0(DISJ) \in \Omega(n)$.

b) Show that its non-deterministic communication complexity satisfies $N^1(DISJ) \in \Omega(n)$.

Problem 3. Let $MED(x, y) : \{0, 1\}^n \times \{0, 1\}^n \to \{1, \ldots, n\}$ be a function that gives the median of the union of the two sets represented by x and y, that is the median of $\{i \in \{1, \ldots, n\};\ x_i = 1 \text{ or } y_i = 1\}$.

a) Show a deterministic protocol for MED which communicates at most $O(\log^2 n)$ bits.

a) Show a deterministic protocol for MED which communicates at most $O(\log n)$ bits.