

Homework Assignment 2 - Decidability and reductions

Deadline: 14.11.2025, 10:40 in Moodle.

Problem 1.

- a) Show that $\leq_m$ is a transitive relation, i.e., if $A \leq_m B$ and $B \leq_m C$ then $A \leq_m C$.
- b) Show that if A is Turing-recognizable and $A \leq_m \overline{A}$, then A is decidable.

Problem 2. Show that for every Turing-recognizable language $A \subseteq \Sigma^*$, $A \leq_m A_{\text{TM}}$.

Problem 3. Consider $L = 0A_{\text{TM}} \cup \overline{1A_{\text{TM}}} = \{0w \in \Sigma^*; w \in A_{\text{TM}}\} \cup \{1w \in \Sigma^*; w \notin A_{\text{TM}}\}$. Is L decidable? Is L Turing-recognizable? Justify your answer.

Problem 4. A state q of a Turing machine M is *useful*, if M enters state q on some input. Show that $L = \{(\langle M \rangle, q); q \text{ is a useful state of a Turing machine } M\}$ is undecidable.