NMMB415 Automata and Computational Complexity **Fall 2024/2025**

Homework Assignment 3 - NP-completeness

Deadline: 2.12.2021, 9:00 in Moodle.

Problem 1. For each function decide and justify whether the relationship is true or false:

a) $n^2 \in o(2n^2)$, $2n^2 \in o(n^3)$

b) $2^n \in o(3^n)$, $2n \in O(n)$

c) $n \log n \in o(n^2)$, $1 \in o(1/n)$

d) $n \log n \in O(2^{3 \log n})$, $2^n \in O(2^{\log^2 n})$

Problem 2. For a set $S \subseteq \{0,1\}^n$ design a Boolean formula $\phi(x_1, x_2, \dots, x_n)$ in CNF form such that for each $a \in \{0,1\}^n$: $\phi(a_1, a_2, \dots, a_n)$ is TRUE if and only if $a \in S$. How large is ϕ relative to n? For bonus points show that for each $n \geq 10$ and some set $S \subseteq \{0,1\}^n$ such a formula must be of size at least $\frac{2^n}{10n}$.

Problem 3. Show that if $P = NP$ then there is an algorithm which for each satisfiable Boolean formula $\phi(x_1, x_2, \dots, x_n)$ finds a satisfying assignment in polynomial time.

Problem 4. Let $LPATH = \{\langle G, s, t, k\rangle$, G is an undirected graph in which s and t are connected by a path of length at least $k\}$. (No vertex can repeat twice on a path.) Show that $LPATH$ is NP-complete.

Problem 5. Let $DOUBLE\text{-}SAT = \{\langle\phi\rangle$, ϕ is a Boolean formula with at least two distinct satisfying assignements$\}$. Show that $DOUBLE\text{-}SAT$ is NP-complete.