NMMB415 Automata and Computational Complexity   **Fall 2024/2025**

Homework Assignment 2 - Decidability and reductions

Deadline: 11.11.2024, 11:11 in Moodle.

Problem 1.

a) Show that $\leq_m$ is a transitive relation, i.e., if $A \leq_m B$ and $B \leq_m C$ then $A \leq_m C$.

b) Show that if A is Turing-recognizable and $A \leq_m \overline{A}$, then A is decidable.

Problem 2.   Show that for every Turing-recognizable language $A \subseteq \Sigma^*$, $A \leq_m A_{\mathrm{TM}}$.

Problem 3.   Consider $L = 0A_{\mathrm{TM}} \cup 1\overline{A_{\mathrm{TM}}} = \{0w \in \Sigma^*; w \in A_{\mathrm{TM}}\} \cup \{1w \in \Sigma^*; w \notin A_{\mathrm{TM}}\}$. Is L decidable? Is L Turing-recognizable? Justify your answer.

Problem 4.   A state q of a Turing machine M is *useless*, if M never enters state q on any input. Show that $L = \{(\langle M \rangle, q); q$ is a useless state of a Turing machine $M\}$ is undecidable.