**NMMB415 Automata and Computational Complexity** **Fall 2021/2022**

**Homework Assignment 3 - $NP$-completeness**

Deadline: 29.11.2021, 12:20 in Moodle.

**Problem 1.** For a set $S \subseteq \{0,1\}^n$ design a Boolean formula $\phi(x_1, x_2, \dots, x_n)$ in CNF form such that for each $a \in \{0,1\}^n$: $\phi(a_1, a_2, \dots, a_n)$ is TRUE if and only if $a \in S$. How large is $\phi$ relative to $n$? For bonus points show that for each $n \geq 10$ and some set $S \subseteq \{0,1\}^n$ such a formula must be of size at least $\frac{2^n}{10n}$.

**Problem 2.** Show that if $P = NP$ then there is an algorithm which for each satisfiable Boolean formula $\phi(x_1, x_2, \dots, x_n)$ finds a satisfying assignment in polynomial time.

**Problem 3.** Let $SPATH = \{\langle G, s, t, k\rangle$, $G$ is an undirected graph in which $s$ and $t$ are connected by a path of length at most $k\}$ and let $LPATH = \{\langle G, s, t, k\rangle$, $G$ is an undirected graph in which $s$ and $t$ are connected by a path of length at least $k\}$. (No vertex can repeat twice on a path.) Show that $SPATH \in P$ and $LPATH$ is $NP$-complete.

**Problem 4.** Let $TRIPLE\text{-}SAT = \{\langle \phi \rangle$, $\phi$ is a Boolean formula with at least three distinct satisfying assignements$\}$. Show that $TRIPLE\text{-}SAT$ is $NP$-complete.