Exercises solved at the recitation on 23. 10. 2007

A *hypergraph* $H = (V, E)$ consists of a set of vertices V and a set of hyperedges E, where each hyperedge is a subset of V (i.e., $E \subseteq 2^V$). A hypergraph is called k*-uniform* if each hyperedge has size k, and it is called r*-regular* if each vertex belongs to r hyperedges. A *bicoloring* of a hypergraph is a coloring of its vertices by two colors, such that every hyperedge contains at least one vertex of each color.

- For $k \geq 3$, show that there is a value $r_0 \equiv r_0(k)$ such that for every $r \leq r_0$ every r-regular k-uniform hypergraph has a bicoloring.
- Try to find a lower bound and an upper bound for the largest possible $r_0(k)$ satisfying the statement above.
- For what values of r can you find an efficient algorithm that finds a bicoloring of a given k-uniform r-regular hypergraph? (An algorithm is considered efficient if its running time is polynomial in the size of the hypergraph, where k and r are considered as constants.)