Úlohy ke cvičení

Úloha 1: Determine, whether the following sets of real functions form a subspace of the vector space of all real functions:

- The polynomials of degree at most 7,
- the polynomials f of degree at most 7 satisfying that $f(0) = 3$,
- the polynomials of degree at most 7 satisfying that -5 and 2 are among their roots,
- the monotone functions,
- the piecewise linear continuous functions.

Yes, no (the zero function is missing), yes, no (two monotone may yield non-monotone), yes.

Úloha 2: Let u, v, w be linear independent vectors in a vector space$V$ over the field $\mathbb{R}$. Decide, whether the following sets of vectors are linearly independent or not.

a) $\{u + v, u - v, u + w, u - w\}$.

$\{u+v, u-v, u+w, u-w\}$ *is linear dependent, for example by the use of coefficients* $(1, 1, -1, -1)^T$.

b) $\{u + v, u + w, v + w\}$.

$\{u+v, u+w, v+w\}$ *is linear independent.*

Úloha 3: Decide, whether the following set of vectors is independent in the arithmetic vector spaces $\mathbb{R}^4, \mathbb{Z}_3^4$ and $\mathbb{Z}_5^4$.

If not, find an expression of some vector as a linear combination of the others.

a) $X_3 = \{(1, 0, 2, 0)^T, (2, 1, 0, 2)^T, (0, 2, 2, 1)^T, (2, 2, 1, 1)^T\}$.

X_3 *is linear independent in* $\mathbb{R}^4$,

it is linear dependent in $\mathbb{Z}_3^4$ *as witnessed by* $(0, 1, 2, 2)^T$,

and it is linear dependent in $\mathbb{Z}_5^4$ *as witnessed by* $(2, 0, 1, 4)^T$.

Úloha 4: Let V be a vector space and $X \subseteq Y \subseteq V$. Decide, which of the following claims are valid or not:

a) The se X is not independent, while the set Y is independent.

Incorrect: $X = \{(1, 0)^T\}$ *and* $Y = \{(1, 0)^T, (0, 1)^T\}$ *and these are both independent in* $\mathbb{R}^2$.

b) If the set X is independent, so is the set Y.

Incorrect: $X = \{(1, 0)^T\}$ *is independent, but* $Y = \{(1, 0)^T, (2, 0)^T\}$ *is dependent in* $\mathbb{R}^2$.

c) If the set Y is independent, o is the set X.

Correct: Every n-tuple of vectors in Y is independent, thus the very same must hold also for all n-tuples of vector in the set X.