Úlohy ke cvičení

Úloha 1: In the field $\mathbb{Z}_5$, solve the matrix equation

$$\begin{pmatrix} 1 & 4 & 2 & 2 & 2 & 3 \\ 4 & 2 & 0 & 0 & 4 & 2 \\ 4 & 2 & 0 & 1 & 1 & 1 \\ 0 & 2 & 4 & 0 & 4 & 4 \\ 3 & 0 & 2 & 2 & 4 & 4 \end{pmatrix} \mathbf{X} = \begin{pmatrix} 1 & 0 & 0 \\ 3 & 2 & 3 \\ 0 & 3 & 4 \\ 0 & 0 & 2 \\ 0 & 1 & 4 \end{pmatrix}$$

Verify the result. You may use Sage, but then provide the commands and intermediate results.

Úloha 2: In the vector space $\mathbb{R}^4$ over the field $\mathbb{R}$ find the linear combination of vectors $(-5, 5, 1, -1)^T$, $(2, -5, 0, 2)^T$, $(3, 2, 0, -2)^T$ a $(2, -3, 1, 1)^T$ which does lead to vector $(-7, 12, 2, -4)^T$. Is this linear combination unique?

Úloha 3: Let $\mathbf{A}$ be a matrix of size $m \times n$ over a field $\mathbb{K}$. Show that $Ker(\mathbf{A})$ forms a vector subspace in the arithmetic vector space $\mathbb{K}^n$.

Úloha 4: Let $\mathbf{D}$ be a square matrix over a field $\mathbb{K}$. Show that all the matrices which commute in matrix product with matrix $\mathbf{D}$ form a vector space.