Úlohy ke cvičení

Úloha 1: Solve the following systems over the field of complex numbers $\mathbb{C}$.

$$
\text{a)} \quad
\begin{array}{ccccccc}
2x_1 & + & (2+2i)x_2 & + & 2ix_3 & = & 1 \\
(1-i)x_1 & + & (1+3i)x_2 & + & (i-1)x_3 & = & 0 \\
(1+i)x_1 & + & (1-i)x_2 & + & (1+i)x_3 & = & 1
\end{array}
$$

Úloha 2: Solve the following system of equations over $\mathbb{Z}_5, \mathbb{Z}_7$ and $\mathbb{R}$.

$$
\begin{array}{ccccccc}
x_1 & + & 2x_2 & + & 4x_3 & = & 3 \\
3x_1 & + & x_2 & + & 2x_3 & = & 4 \\
2x_1 & + & 4x_2 & + & x_3 & = & 3
\end{array}
$$

Úloha 3: For $n \in \mathbb{N}$ and an asociative operation $\cdot$ let $a^n = a \cdot a \cdot \ldots \cdot a$, where the element a appears n times in the product.

a) Determine values $2^{101}, 3^{1\,001}$ and $4^{1\,000\,001}$ in the field $\mathbb{Z}_{17}$.

Úloha 4: Invert the following matrices over fields $\mathbb{Z}_3$ and $\mathbb{Z}_5$

a) $\mathbf{C} = \begin{pmatrix} 2 & 0 & 1 & 0 \\ 1 & 2 & 0 & 1 \\ 2 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \end{pmatrix}$.

b) $\mathbf{D} = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 2 & 0 \\ 0 & 1 & 2 & 1 \end{pmatrix}$.