Úlohy ke cvičení

Úloha 1: In the vector space of real functions, determine whether the following polynomials are linearly independent or not.

$x^5 + 7x^3 - 9x^2 + 2x + 3$, $-x^5 + x^4 - 5x^3 + 3x^2 - 6x - 8$, $-x^5 + x^4 - 5x^3 + 4x^2 - 4x - 4$, $2x^4 + 4x^3 - 6x^2 + 4x + 14$, and $3x^5 - 7x^4 + 7x^3 + 5x^2 + 14x + 4$.

If they are linearly dependent, express some of them as a linear combination of others.

You may use Sage.

Úloha 2: Extend the set M to a basis of the vector space V

a) $M = \{(1, 2, 0, 0)^T, (2, 1, 1, 3)^T, (0, 1, 0, 1)^T\}$, $V = \mathbb{R}^4$.

b) $M = \{-x^2, x + x^2, x^3 - 1\}$, in the space V of real polynomes of degree at most three.

Úloha 3: Determine dimensions and bases of the following subspaces of $\mathbb{Z}_5^7$.

a) $U_1 = \mathcal{L}((4, 1, 0, 3, 4, 0, 0)^T, (4, 3, 1, 0, 2, 3, 1)^T, (4, 1, 4, 0, 3, 2, 4)^T, (2, 4, 1, 4, 4, 3, 1)^T, (0, 4, 3, 2, 2, 4, 3)^T)$.

b) $V_1 = \{(x_1, \ldots, x_7)^T \in \mathbb{Z}_5^7 : x_1 + 3x_2 + x_3 + 2x_4 + 3x_5 + x_6 + 2x_7 = 0, 3x_1 + 4x_2 + 3x_3 + x_4 + 4x_5 + 2x_6 + 4x_7 = 0,\ 2x_1 + x_2 + 4x_3 + 4x_5 + 2x_7 = 0\}$.

Úloha 4: Determine, whether the spaces U_i and V_i are in an inclusion. If so, find a basis of the larger one that extend a basis of the smaller one.

These subspaces of $\mathbb{Z}_5^7$ are defined as follows:

a) $U_1 = \mathcal{L}((4, 1, 0, 3, 4, 0, 0)^T, (4, 3, 1, 0, 2, 3, 1)^T, (4, 1, 4, 0, 3, 2, 4)^T, (2, 4, 1, 4, 4, 3, 1)^T, (0, 4, 3, 2, 2, 4, 3)^T)$

$V_1 = \{(x_1, \ldots, x_7)^T \in \mathbb{Z}_5^7 : x_1 + 3x_2 + x_3 + 2x_4 + 3x_5 + x_6 + 2x_7 = 0, 3x_1 + 4x_2 + 3x_3 + x_4 + 4x_5 + 2x_6 + 4x_7 = 0,\ 2x_1 + x_2 + 4x_3 + 4x_5 + 2x_7 = 0\}$