Discrete Mathematics

Exercise sheet 10

5/ 13 December 2016

1. Prove that the complement of a disconnected graph G is connected. (The complement of a graph $G = (V, E)$ is the graph $(V, \binom{V}{2} \backslash E)$.)

What is the complement of the disjoint union of two complete graphs K_m and K_n?

[Matoušek & Nešetřil, *Invitation to Discrete Mathematics*, section 4.2.]

2. Let $G = (V, E)$ be a graph.

(a) Define what is meant by a *subgraph* of G and by an *induced subgraph* of G.

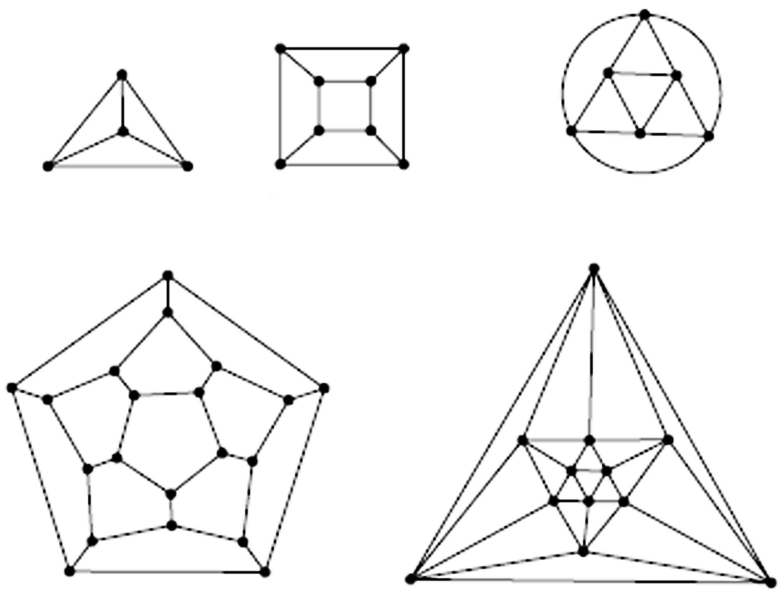
(b) Show that if G contains an odd cycle as a subgraph then it also contains an odd cycle as an induced subgraph.

(c) Give a counterexample to the statement that if G contains an even cycle as a subgraph then it also contains an even cycle as an induced subgraph.

3. A *Hamiltonian cycle* in a graph G is a cycle containing all vertices of G.

(Write down the definition of an Eulerian tour and see how this differs from the notion of a Hamiltonian cycle.)

(a) Decide which of the graphs in the figure has a Hamiltonian cycle.

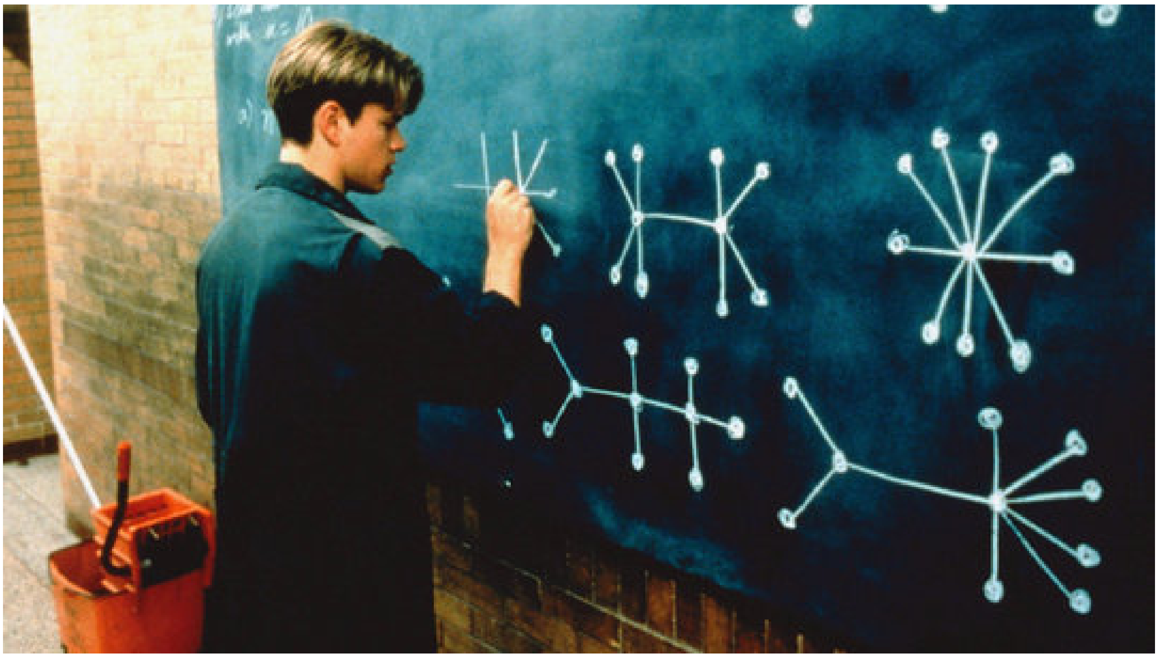


(b) Construct two connected graphs with the same score such that one has a Hamiltonian cycle while the other one does not.

[Matoušek & Nešetřil, *Invitation to Discrete Mathematics*, section 4.4.]

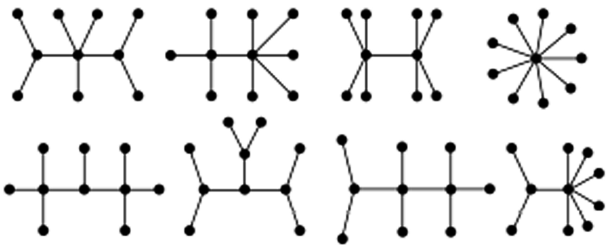
4. A *tree* is a connected graph containing no cycles as a subgraph.

A tree is *homeomorphically irreducible* if it has no vertices of degree 2. (So a path on 3 or more more vertices is not homeomorphically irreducible.)



One of the problems in the film *Good Will Hunting* (1997) is to find all homeomorphically irreducible trees with 10 vertices.

The filmmakers made a mistake: Will draws just 8 trees on the board, while in fact there exist 10 homeomorphically irreducible trees with 10 vertices. Here are the ones he draws on the board:



Find the two homeomorphically irreducible trees on 10 vertices that Will misses.