A ϕ-Competitive Algorithm for Scheduling Packets with Deadlines

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Joint work with Marek Chrobak (UC Riverside), Łukasz Jeż (Wrocław), and Jiří Sgall (Charles University, Prague)

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Outline

- Introduction to competitive analysis
- Model & result
- Algorithm
- Analysis techniques
- Further research directions

Introduction to competitive analysis

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These are worst-case scenarios

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Online computation

-

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-

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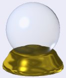

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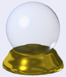

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- Sequence of *events* (orders), arrive over time

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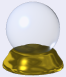

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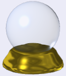

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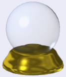

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- Some events ask to make *decisions* (Monday mornings)
- Decisions influence the *objective function* (rewards served orders)

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- Worst-case ratio between
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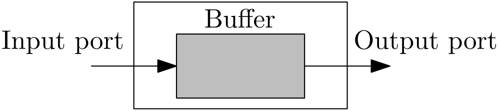
- Game: the algorithm vs. an adversary 
  - The adversary decides on further input to maximize OPT/ALG

Model & Result

Online Packet Scheduling with Deadlines

Network switch:



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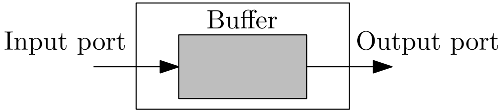

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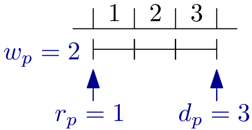

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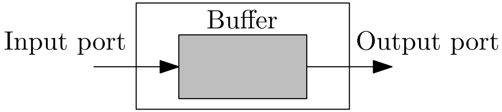

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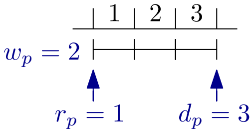

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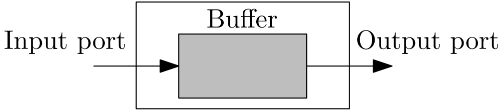

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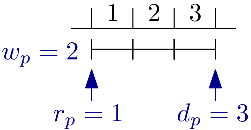

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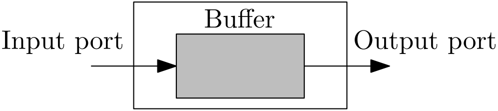

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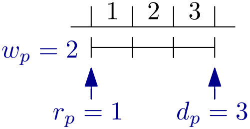

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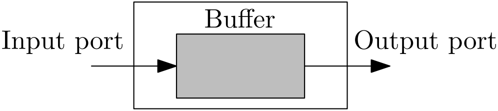
Network switch: 

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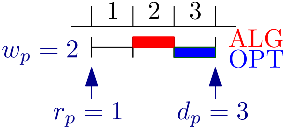

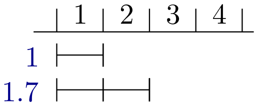

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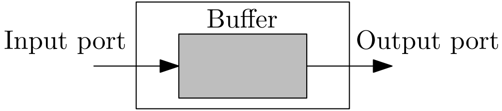

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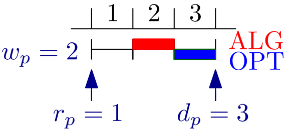

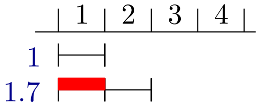

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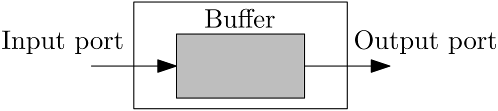

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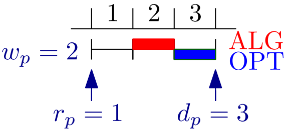

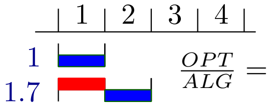

$$\frac{OPT}{ALG} = \frac{1.7+1}{1.7} \approx 1.59$$

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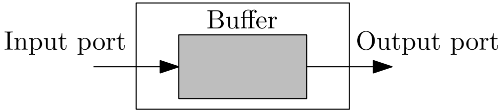

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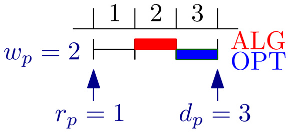

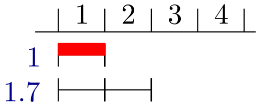

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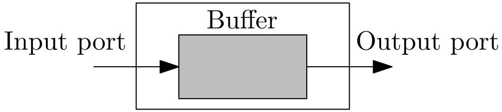

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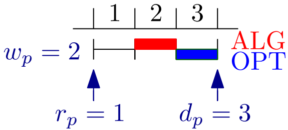

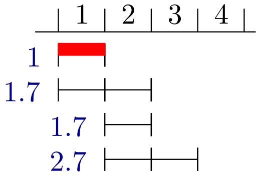

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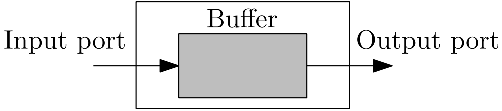

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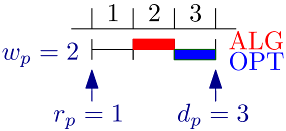

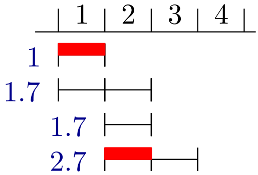

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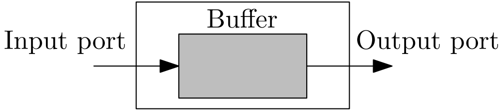

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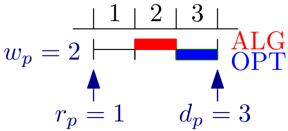

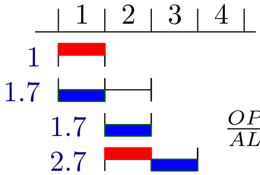

$$\frac{OPT}{ALG} = \frac{2\cdot 1.7 + 2.7}{1.7 + 2.7} \approx 1.39$$

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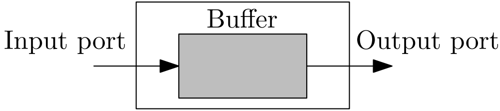

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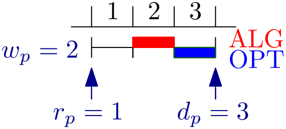

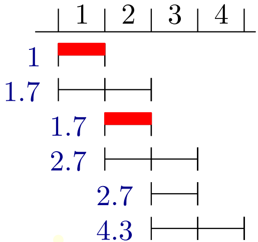

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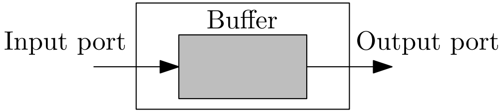

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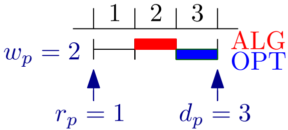

Scheduling problem $1|online, r_j, p_j = 1| \sum w_j(1 - U_j)$

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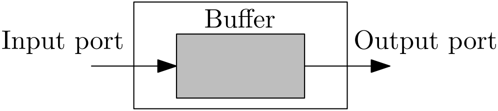

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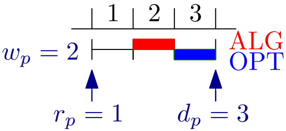

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A.k.a. Buffer Management in Quality of Service Switches

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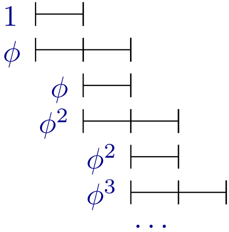

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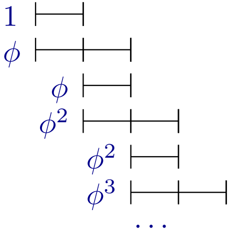

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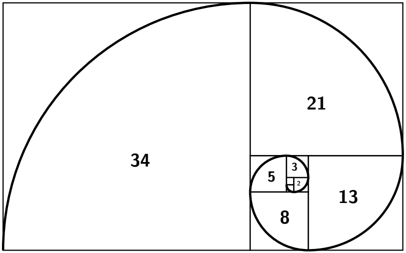

Previous work

- We focus on deterministic algorithms
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  - Schedules always the heaviest pending packet
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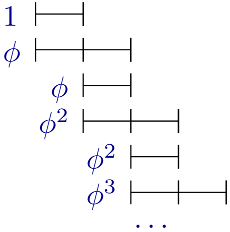

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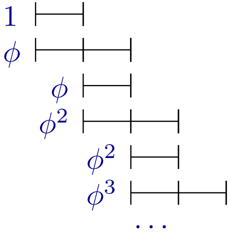

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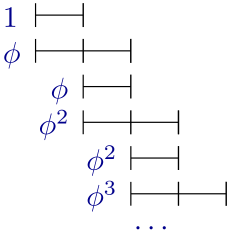

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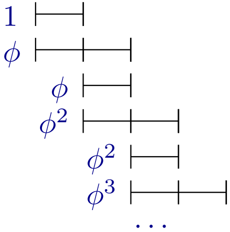

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New result

Theorem

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- Optimal future profit unless new packets arrive
- Scheduled plans (a.k.a. provisional schedules) used already by [Li *et al.* '05, Li *et al.* '07, Englert & Westermann '07]

Algorithm

Algorithm $\textsc{Plan}(\phi)$

Plan $\mathcal{P}$

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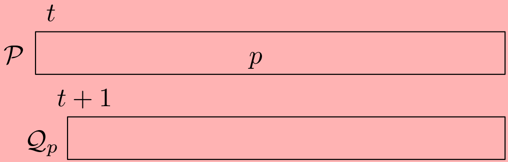

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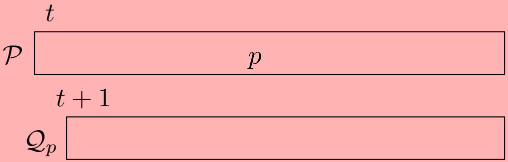

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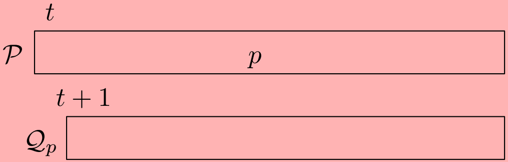

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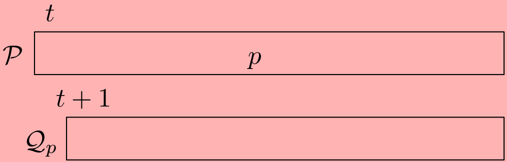

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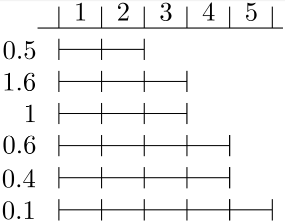

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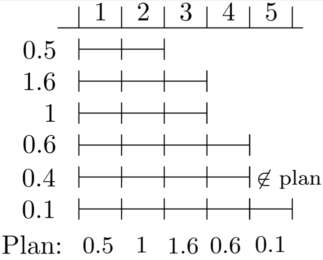

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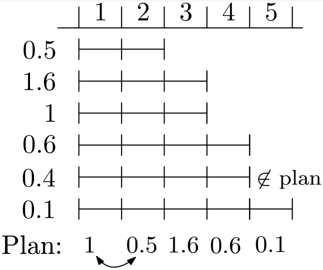

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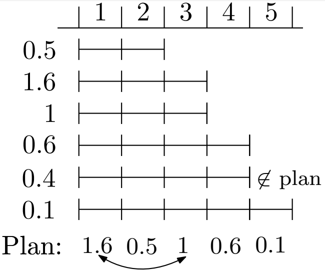

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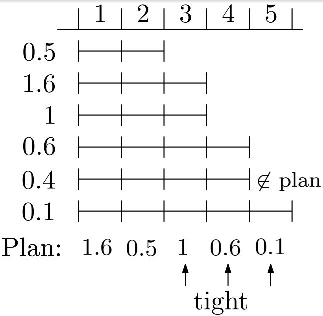

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Slot τ is *tight* w.r.t. plan $\mathcal{P}$ iff

# of slots till τ = # of packets $j \in \mathcal{P} : d_j \leq \tau$

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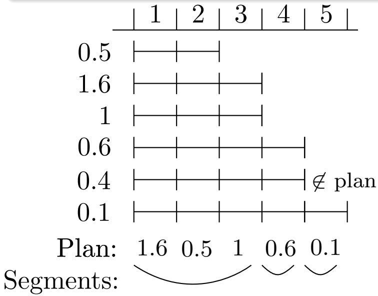

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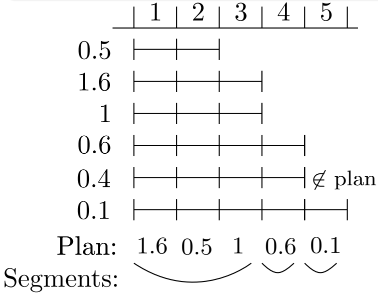

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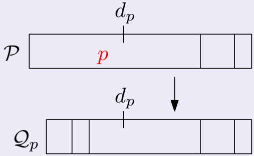
Plan Updates After Packet p is Scheduled

p in the 1st segment ("greedy step")



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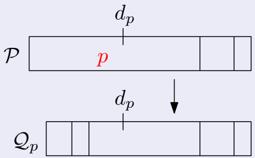

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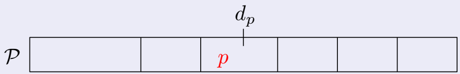

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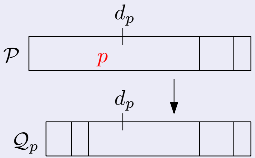

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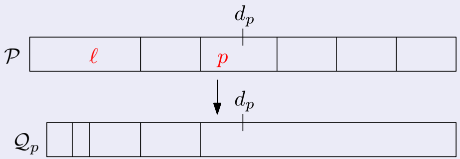

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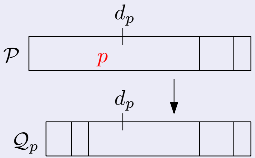

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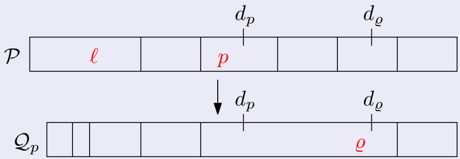

- ℓ = lightest in the 1st segment
- ϱ = heaviest not in $\mathcal{P}$ which can replace p
  - *replacement packet* for p

Problem of $\textsc{Plan}(\phi)$: Weight Decreases in the Plan

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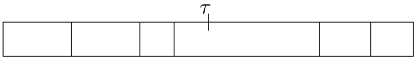

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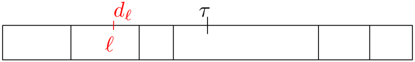

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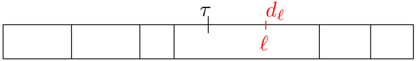

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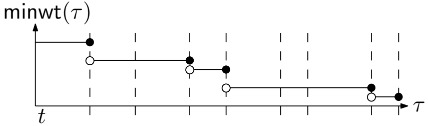

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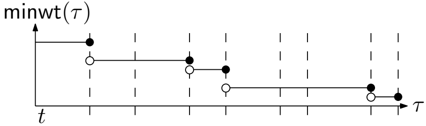

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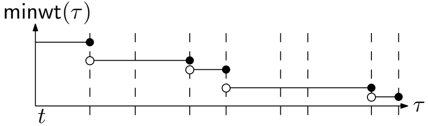

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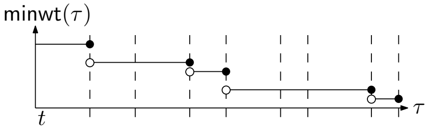

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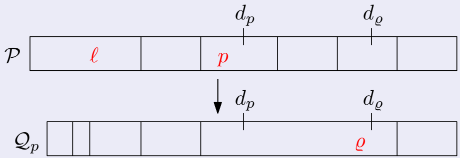

The problem:

$\varrho \notin \mathcal{P} \Rightarrow w_\varrho < \text{minwt}(d_\varrho)$

Solution: Maintaining Slot-Monotonicity of minwt

- Idea: modify $\textsc{Plan}(\phi)$ so that $\mathrm{minwt}(\tau)$ never decreases for any τ

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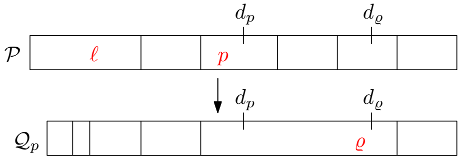

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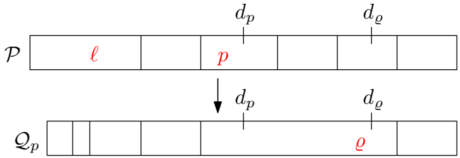

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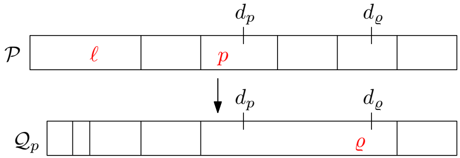

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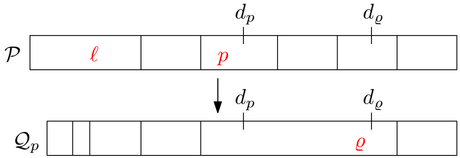

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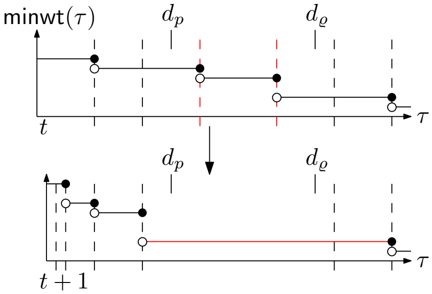

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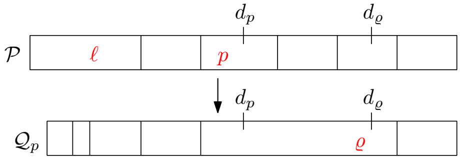

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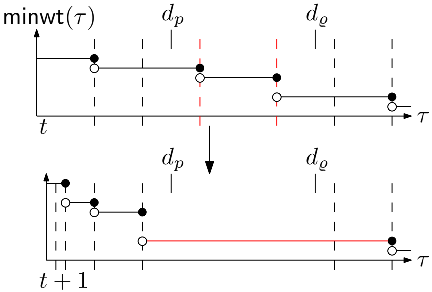

- $\Rightarrow$ avoid merging segments

Algorithm $\textsc{PlanM}(\phi)$ Maintaining Slot-Monotonicity

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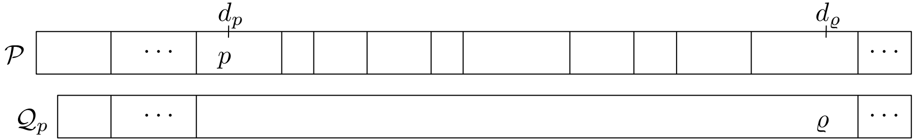

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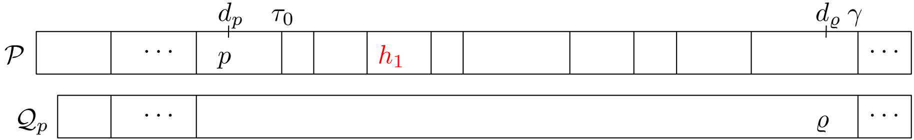

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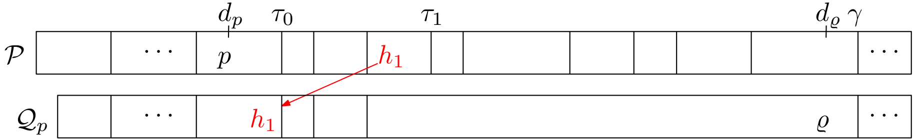

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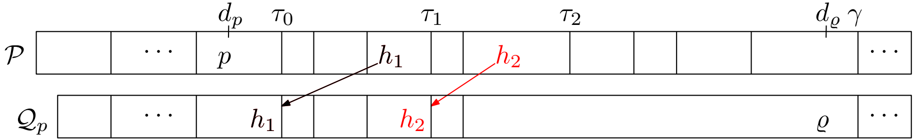

  - h_2 = heaviest packet in $(\tau_1, \gamma]$,
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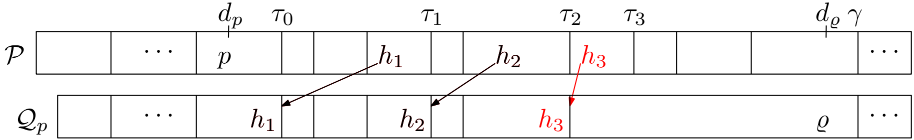

  - h_3 = heaviest packet in $(\tau_2, \gamma]$,
  - decrease deadline of h_3 to τ_2

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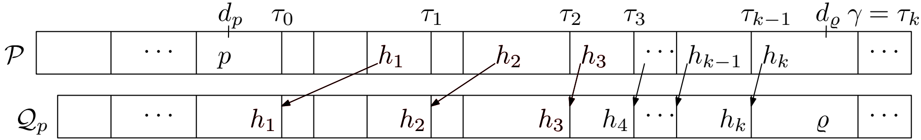

  - for $i = 1, 2, \ldots$: h_i = heaviest packet in $(\tau_{i-1}, \gamma]$,
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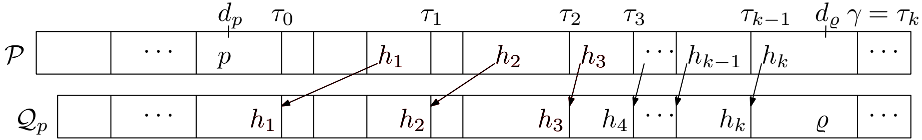

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In a nutshell

Avoid merging segments and minwt decreases in a right way

- Done by decreasing deadlines and increasing weights of certain packets

Analysis

Analysis Overview

- Competitive analysis
  - Goal: $w(\mathrm{OPT}) \leq \phi \cdot w(\mathrm{ALG})$ for any instance

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2. Potential function
3. Modifications of the adversary (optimal) schedule ADV

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Invariant (A)

ADV consists of two types of packets:
- (real) packets in plan P
- all other packets are treasures 

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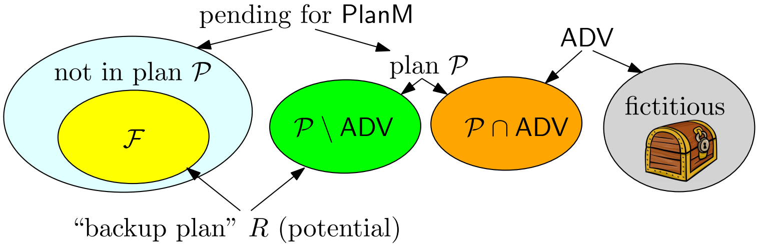
$$\Psi := \frac{1}{\phi}\, w(R)$$

Packet Types in the Analysis



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Conclusions

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$\phi \approx 1.618$-competitive deterministic algorithm

- Schedule packet $p \in \mathcal{P}$ maximizing $\phi \cdot w_p + w(\mathcal{Q}_p)$
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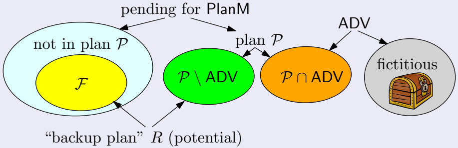

Further Research Directions

$m \geq 1$ packets are sent in each step

- Our algorithm is $\phi \approx 1.618$-competitive for any $m \geq 1$
- The best algorithm has ratio $\frac{1}{1-\left(\frac{m}{m+1}\right)^m} \to \frac{e}{e-1} \approx 1.58$ [Chin *et al.* '04]
- Can our algorithm be modified to give a better ratio for $m > 1$?

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