Online Packet Scheduling with Bounded Delay and Lookahead

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[4]George Mason University, USA.

MAPSP 2017, June 12

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- Unit-length packets arrive over time
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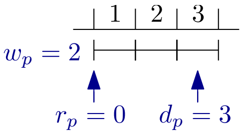

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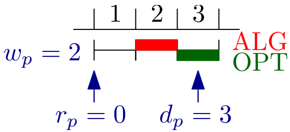

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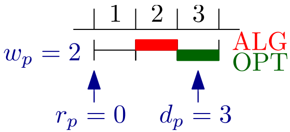

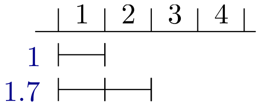

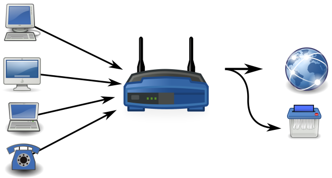

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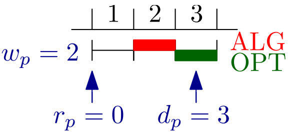

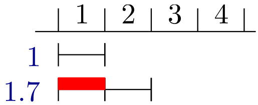

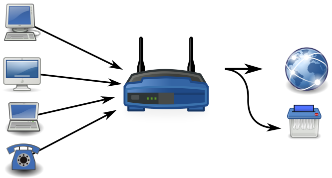

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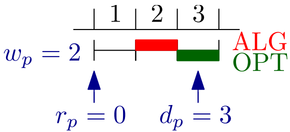
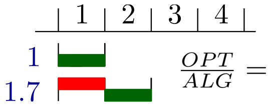
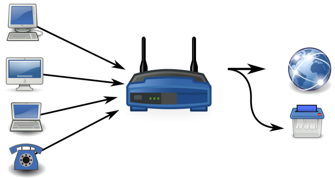




$$\frac{OPT}{ALG} = \frac{1.7+1}{1.7} \approx 1.59$$



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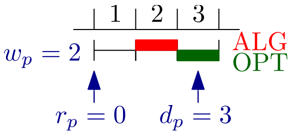

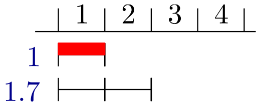

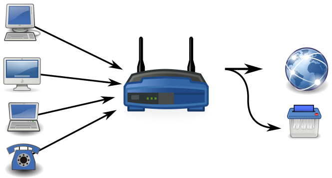

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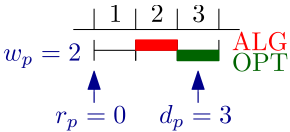

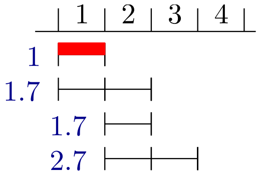

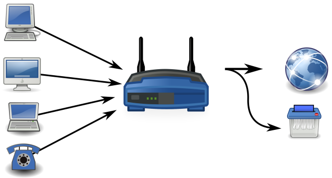

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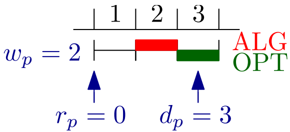

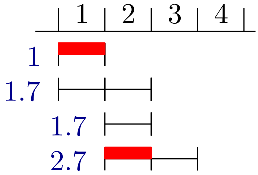

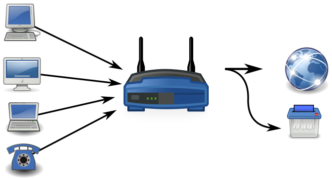

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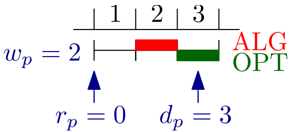
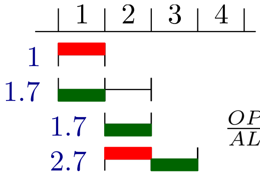
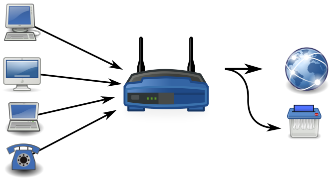




$$\frac{OPT}{ALG} = \frac{2\cdot 1.7+2.7}{1.7+2.7} \approx 1.39$$



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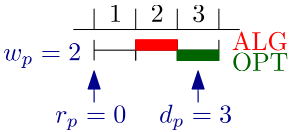
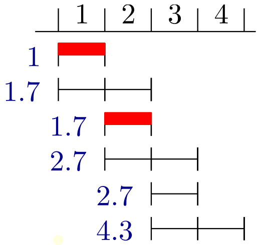
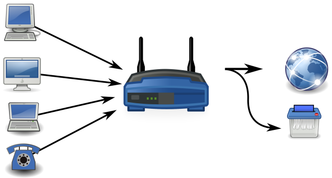






Competitive ratio of online algorithms

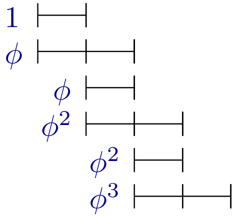
- ALG is R-competitive if for any instance I

$$ALG(I) \geq \frac{1}{R} OPT(I)$$

Previous work

- We focus on deterministic algorithms
- Lower bound of the golden ratio $\phi = \frac{1}{2}(\sqrt{5}+1) \approx 1.618$

$$1 + \frac{1}{\phi} = \phi$$



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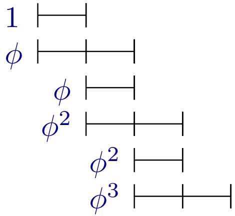

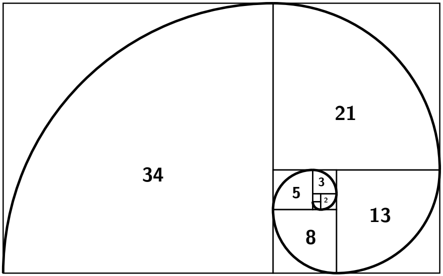

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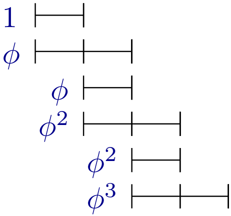
- $2\sqrt{2} - 1 \approx 1.828$-competitive algorithm by Englert and Westermann



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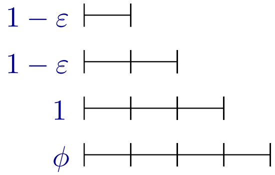

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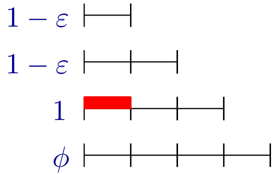

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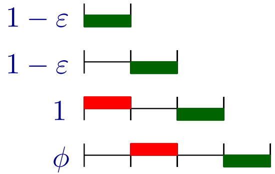

Our results

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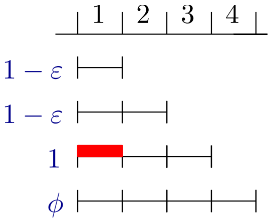
- New model with *lookahead*
  - ℓ-lookahead = at time t algorithm sees packets arriving by time $t + \ell$
  - Deterministic algorithms for 2-bounded instances
  - 1.303-competitive algorithm with 1-lookahead
  - Lower bound for ℓ-lookahead

ToggleH: algorithm for 4-bounded instances

- Modification of EDF_ϕ



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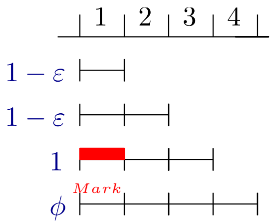

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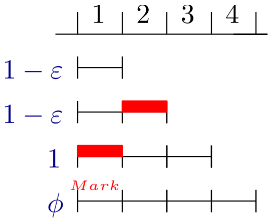

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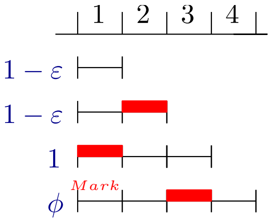

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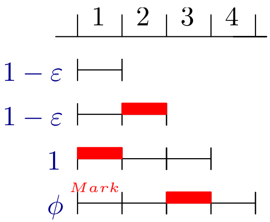

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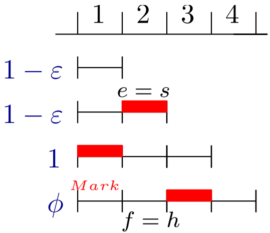
- s = the second-heaviest packet
- e = the earliest-deadline packet with $w_e \geq \frac{1}{\phi^2} w_h$



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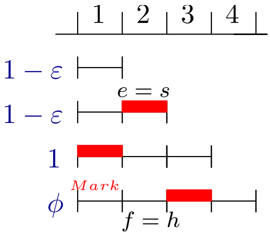

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if (h marked in the previous step) $\wedge$ ($w_s < w_h/\phi$) $\wedge$ ($d_e = t$)
  schedule e

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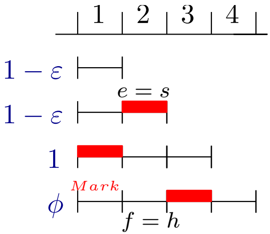

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    schedule e
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    schedule f
    if ($d_h = t+3$) $\wedge$ ($d_f = t+2$) **then** mark h

Charging scheme

- Idea: assign the weight of each packet in an optimal schedule to slots in algorithm's schedule.
- Goal: no slot t receives more that ϕw_{p_t}
  - p_t is the packet scheduled at t in ALG.

ALG	p_1	p_2	p_3	
OPT	p_0	p_2	p_1	p_3

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if (h marked in the previous step) $\wedge$ ($w_s < w_h/\phi$) $\wedge$ ($d_e = t$)
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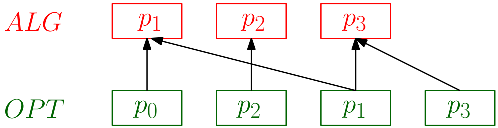

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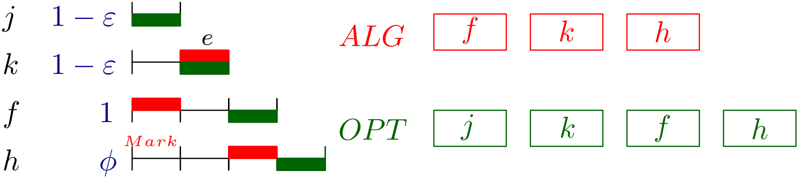

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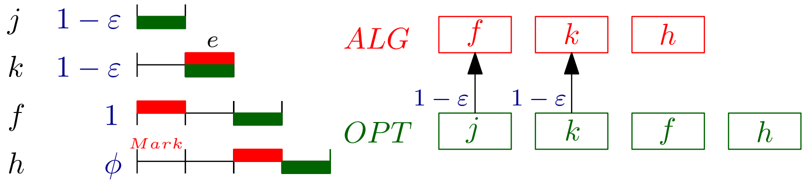

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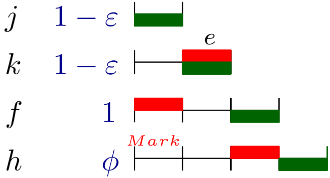

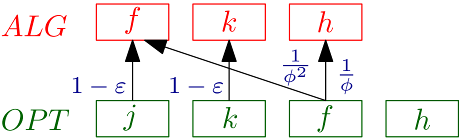

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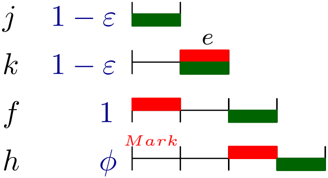

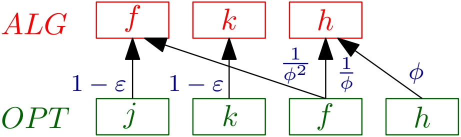

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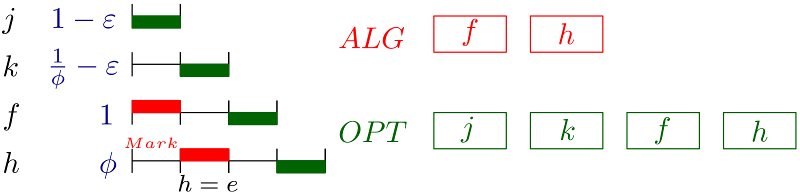

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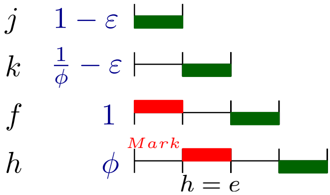

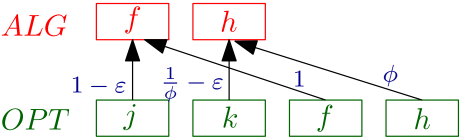

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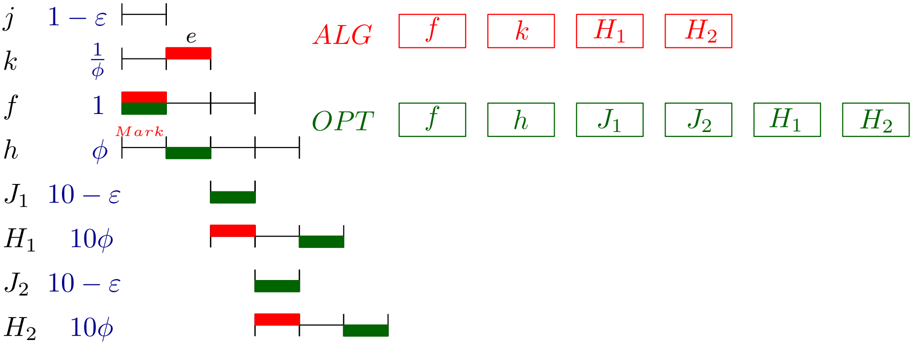

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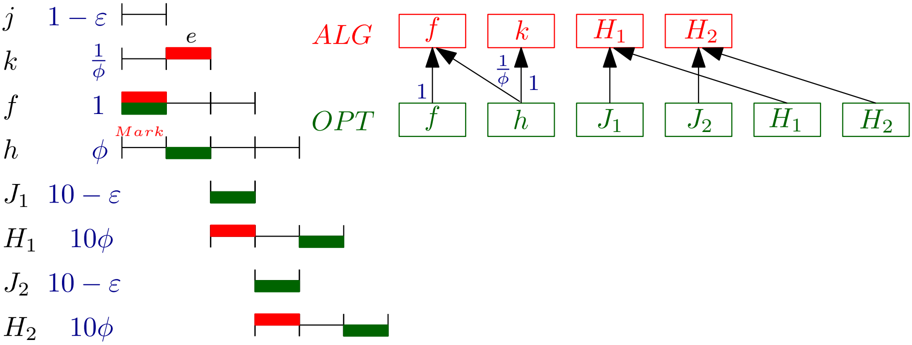

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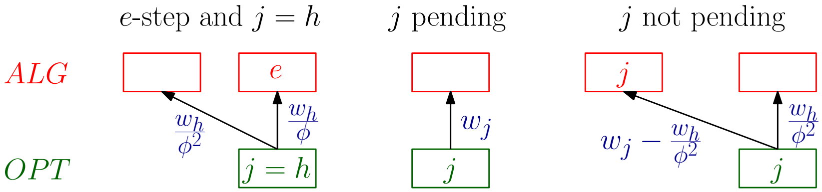

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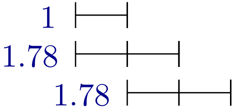

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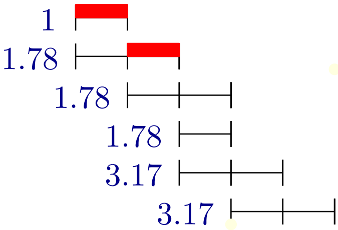

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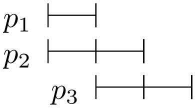

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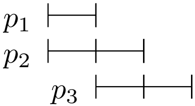

CompareWithBias(α)

if $r_{p_2} = t$ **and** $w_{p_1} < \min\left(w_{p_2}, w_{p_3}, \frac{1}{2\alpha}(w_{p_2} + w_{p_3})\right)$
  then schedule p_2
else schedule p_1

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  - Or find a better lower bound with large span

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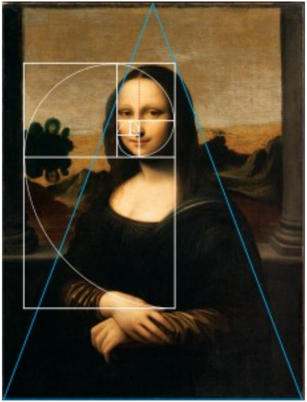

Thank you!

Randomized algorithms

Against oblivious adversary:

	General	2-bounded	s-bounded
Lower bound	1.25	1.25	1.25
Upper bound	$\frac{e}{e-1} \approx 1.582$	1.25	$\frac{1}{1-(1-\frac{1}{s})^s}$

Against adaptive adversary:

	General	2-bounded	s-bounded
Lower bound	1.333	1.333	1.333
Upper bound	$\frac{e}{e-1} \approx 1.582$	1.333	$\frac{1}{1-(1-\frac{1}{s})^s}$