Úlohy ke cvičení

Úloha 1: Complete the chain of elementary transformations s. t. the matrices contain only integers:

$$\begin{pmatrix} 6 & 2 & 4 & 2 \\ 7 & -1 & 2 & 2 \\ -8 & 4 & 0 & -2 \end{pmatrix} \sim \begin{pmatrix} 3 & . & . & . \\ 7 & . & . & . \\ -8 & . & . & . \end{pmatrix} \sim \begin{pmatrix} 3 & . & . & . \\ 7 & . & . & . \\ 4 & . & . & . \end{pmatrix} \sim \begin{pmatrix} 3 & . & . & . \\ 4 & . & . & . \\ 4 & . & . & . \end{pmatrix} \sim$$

$$\sim \begin{pmatrix} 3 & . & . & . \\ 4 & . & . & . \\ 0 & . & . & . \end{pmatrix} \sim \begin{pmatrix} 3 & . & . & . \\ 1 & . & . & . \\ 0 & . & . & . \end{pmatrix} \sim \begin{pmatrix} 0 & . & . & . \\ 1 & . & . & . \\ 0 & . & . & . \end{pmatrix} \sim \begin{pmatrix} 1 & . & . & . \\ 0 & . & . & . \\ 0 & . & . & . \end{pmatrix}$$

Úloha 2: By use of elementary transformations, reduce the following matrix to the echelon form.

a) $\begin{pmatrix} 2 & 3 & 4 & 5 \\ 3 & 0 & 2 & -3 \\ 7 & 6 & 10 & 7 \end{pmatrix}$

b) $\begin{pmatrix} 2 & -3 & 13 & 18 \\ 6 & -9 & 7 & 10 \\ 2 & -3 & -3 & -4 \end{pmatrix}$

c) $\begin{pmatrix} 1 & -1 & 1 & 2 \\ 1 & 8 & 7 & -7 \\ 1 & 2 & 3 & -1 \\ 1 & 5 & 5 & -4 \end{pmatrix}$

Úloha 3: By elementary transformations decide which of the following systems of equations have the same sets of solutions. It is not necessary to compute these sets.

$$\begin{aligned} x_1 + 3x_2 \qquad\quad + 4x_4 &= 0 \\ 2x_1 + x_2 + 3x_3 \qquad &= 0 \\ 3x_1 \qquad + x_3 + 2x_4 &= 0 \end{aligned} \quad \text{(A)}$$

$$\begin{aligned} x_1 - 2x_2 + 3x_3 - 4x_4 &= 0 \\ x_1 + 3x_2 \qquad\quad + 4x_4 &= 0 \\ 9x_1 \qquad + 3x_3 + 6x_4 &= 0 \end{aligned} \quad \text{(B)}$$

$$\begin{aligned} 3x_1 + 4x_2 + 3x_3 + 4x_4 &= 0 \\ x_1 + 3x_2 \qquad\quad + 4x_4 &= 0 \\ 2x_1 + x_2 + 3x_3 \qquad &= 0 \end{aligned} \quad \text{(C)}$$

$$\begin{aligned} 3x_1 \qquad + x_3 + 2x_4 &= 0 \\ 3x_1 + 9x_2 \qquad\quad + 12x_4 &= 0 \\ 3x_1 + 4x_2 + 3x_3 + 4x_4 &= 0 \end{aligned} \quad \text{(D)}$$

$$\begin{aligned} 2x_1 + 6x_2 \qquad\quad + 8x_4 &= 0 \\ x_1 - x_2 - 2x_3 + 2x_4 &= 0 \\ 4x_1 + 3x_2 + 1x_3 + 6x_4 &= 0 \\ 2x_1 + x_2 + 3x_3 \qquad &= 0 \end{aligned} \quad \text{(E)}$$

$$\begin{aligned} 4x_1 + 2x_2 + 6x_3 \qquad &= 0 \\ 3x_1 \qquad + x_3 + 2x_4 &= 0 \\ 3x_1 - x_2 + 6x_3 + 4x_4 &= 0 \end{aligned} \quad \text{(F)}$$

$$\begin{aligned} x_1 + 3x_2 \qquad\quad + 4x_4 &= 0 \\ - 5x_2 + 3x_3 - 8x_4 &= 0 \\ - 9x_2 + x_3 - 10x_4 &= 0 \end{aligned} \quad \text{(G)}$$

$$\begin{aligned} 6x_1 \qquad + 2x_3 + 4x_4 &= 0 \\ x_2 - 5x_3 - 2x_4 &= 0 \\ 2x_1 + x_2 + 3x_3 \qquad &= 0 \end{aligned} \quad \text{(H)}$$